Tracer Budgets in ECCO—Part II: Technical Details and Some Examples

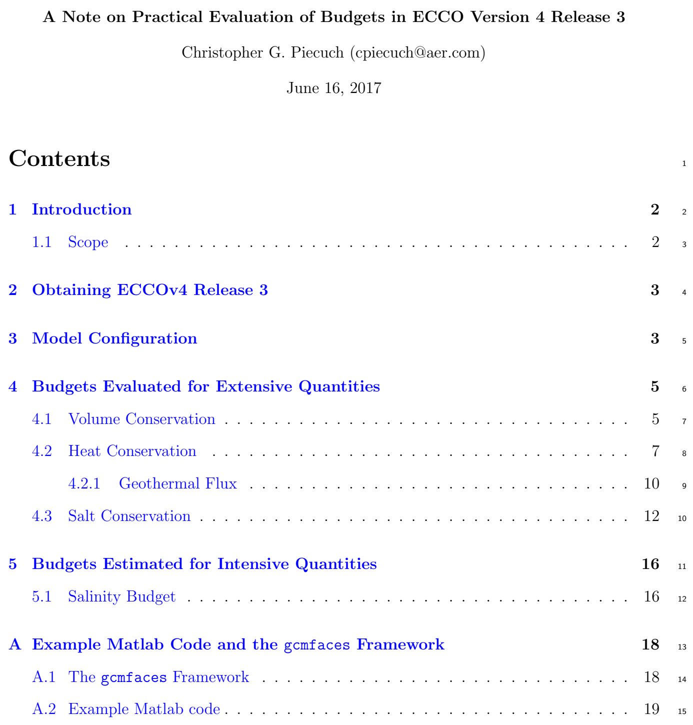
Christopher G. Piecuch (cpiecuch@whoi.edu)
Woods Hole Oceanographic Institution, Woods Hole, MA

Estimating the Circulation and Climate of the Ocean (ECCO) Summer School
May 19-31, 2019 | Friday Harbor Laboratories | University of Washington

Salt, Volume, & Salinity Budgets

A Practical Guide

- A full guide is available and downloadable—
  - https://dspace.mit.edu/handle/1721.1/111094



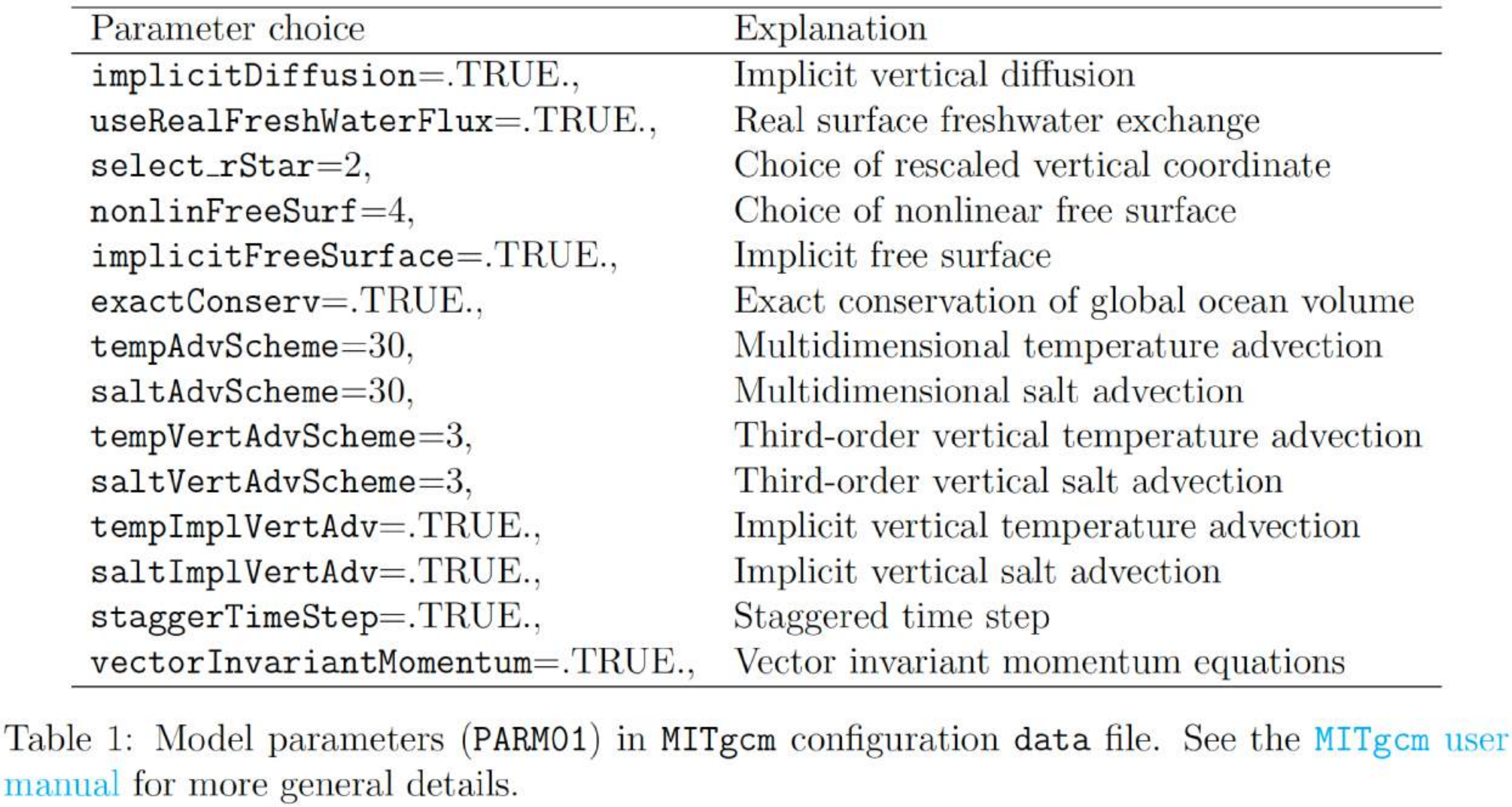
A Note on Practical Evaluation of Budgets in ECCO Version 4 Release 3

Christopher G. Piecuch (cpiecuch@aer.com)

June 16, 2017

Contents

Pay Attention to Details of Model Setup!



Parameter choice	Explanation
`implicitDiffusion`=.TRUE.,	Implicit vertical diffusion
`useRealFreshWaterFlux`=.TRUE.,	Real surface freshwater exchange
`select_rStar`=2,	Choice of rescaled vertical coordinate
`nonlinFreeSurf`=4,	Choice of nonlinear free surface
`implicitFreeSurface`=.TRUE.,	Implicit free surface
`exactConserv`=.TRUE.,	Exact conservation of global ocean volume
`tempAdvScheme`=30,	Multidimensional temperature advection
`saltAdvScheme`=30,	Multidimensional salt advection
`tempVertAdvScheme`=3,	Third-order vertical temperature advection
`saltVertAdvScheme`=3,	Third-order vertical salt advection
`tempImplVertAdv`=.TRUE.,	Implicit vertical temperature advection
`saltImplVertAdv`=.TRUE.,	Implicit vertical salt advection
`staggerTimeStep`=.TRUE.,	Staggered time step
`vectorInvariantMomentum`=.TRUE.,	Vector invariant momentum equations

Table 1: Model parameters (`PARM01`) in `MITgcm` configuration `data` file. See the `MITgcm` user manual for more general details.

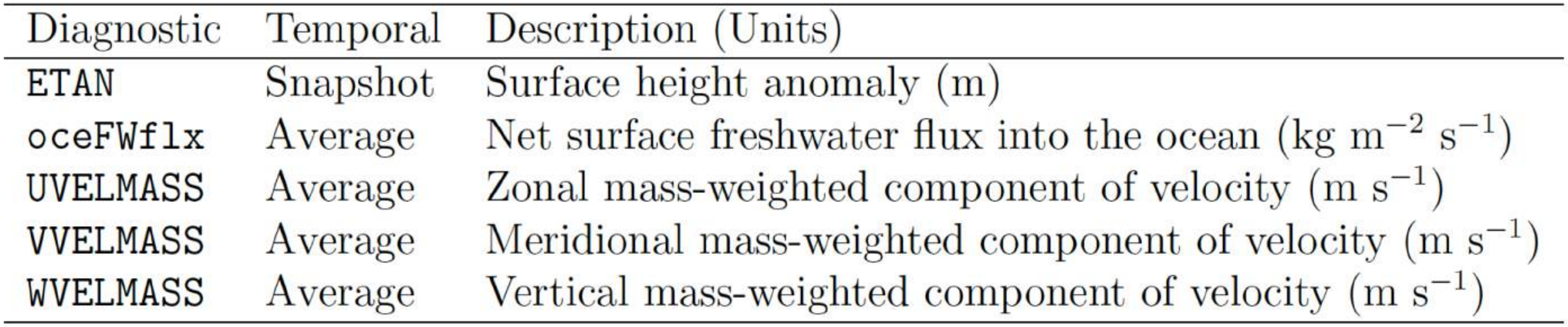
Discretizing the Primitive Equations: Volume

$$\underbrace{\frac{1}{H}\frac{\partial\eta}{\partial t}}_{G^{\eta,tot}} = \underbrace{-\nabla_{z^*}\left(s^*\mathbf{v}\right) - \frac{\partial w}{\partial z^*}}_{G^{\eta,conv}} + \underbrace{s^*\mathcal{F}}_{G^{\eta,forc}}$$

$$\frac{1}{H}\frac{\eta^{n+1}-\eta^{n}}{\Delta t} = -\nabla_{z^*}\left(s^{*n}\mathbf{v}^{n+1}\right) - \frac{\partial w^{n+1}}{\partial z^*} + s^{*n}\mathcal{F}^{n+1/2}$$

Forget et al. (2015), Geosci. Mod. Dev.

Discretizing the Primitive Equations: Volume

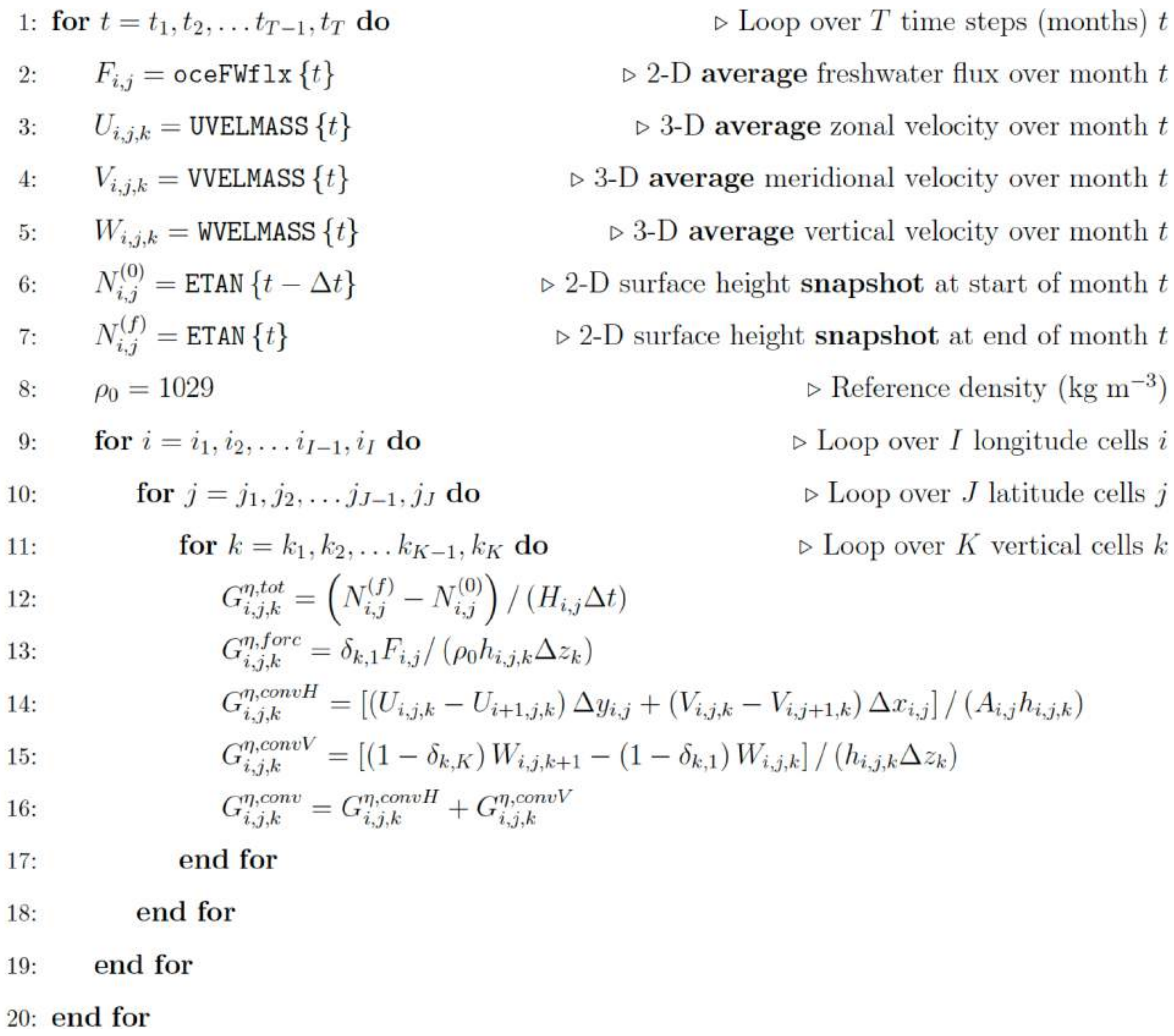


Diagnostic	Temporal	Description (Units)
`ETAN`	Snapshot	Surface height anomaly (m)
`oceFWflx`	Average	Net surface freshwater flux into the ocean (kg m^{-2} s^{-1})
`UVELMASS`	Average	Zonal mass-weighted component of velocity (m s^{-1})
`VVELMASS`	Average	Meridional mass-weighted component of velocity (m s^{-1})
`WVELMASS`	Average	Vertical mass-weighted component of velocity (m s^{-1})

Table 2: `MITgcm` diagnostics required to evaluate the vertically integrated volume budget.

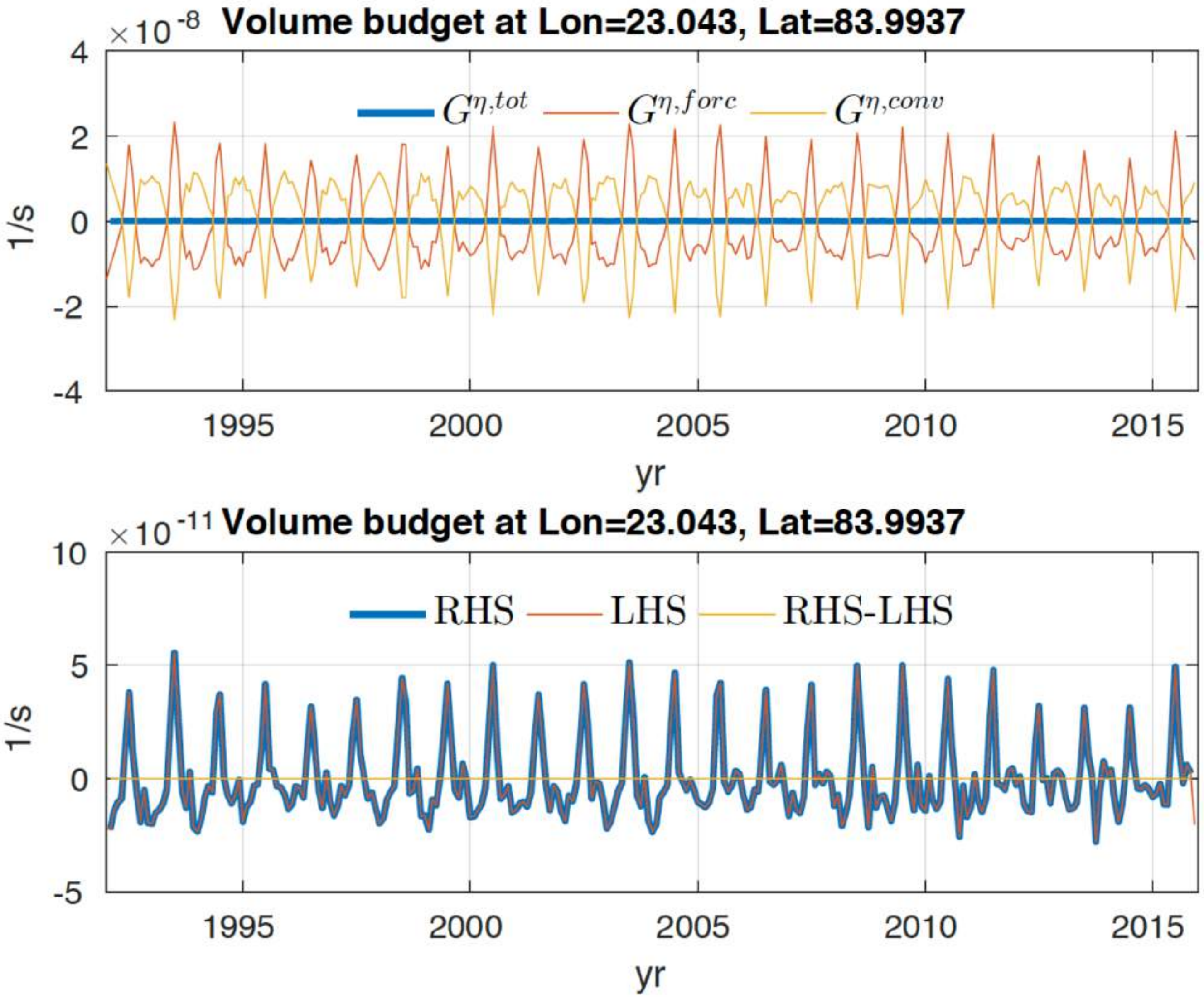
Discretizing the Primitive Equations: Volume

1.



```
for $t = t_1, t_2, \dots t_{T-1}, t_T$ do    $\triangleright$ Loop over $T$ time steps (months) $t$
    $F_{i,j} = \texttt{oceFWflx}\,\{t\}$    $\triangleright$ 2-D **average** freshwater flux over month $t$
    $U_{i,j,k} = \texttt{UVELMASS}\,\{t\}$    $\triangleright$ 3-D **average** zonal velocity over month $t$
    $V_{i,j,k} = \texttt{VVELMASS}\,\{t\}$    $\triangleright$ 3-D **average** meridional velocity over month $t$
    $W_{i,j,k} = \texttt{WVELMASS}\,\{t\}$    $\triangleright$ 3-D **average** vertical velocity over month $t$
    $N_{i,j}^{(0)} = \texttt{ETAN}\,\{t - \Delta t\}$    $\triangleright$ 2-D surface height **snapshot** at start of month $t$
    $N_{i,j}^{(f)} = \texttt{ETAN}\,\{t\}$    $\triangleright$ 2-D surface height **snapshot** at end of month $t$
    $\rho_0 = 1029$    $\triangleright$ Reference density (kg m$^{-3}$)
    for $i = i_1, i_2, \dots i_{I-1}, i_I$ do    $\triangleright$ Loop over $I$ longitude cells $i$
        for $j = j_1, j_2, \dots j_{J-1}, j_J$ do    $\triangleright$ Loop over $J$ latitude cells $j$
            for $k = k_1, k_2, \dots k_{K-1}, k_K$ do    $\triangleright$ Loop over $K$ vertical cells $k$
                $G_{i,j,k}^{\eta,tot} = \left(N_{i,j}^{(f)} - N_{i,j}^{(0)}\right) / \left(H_{i,j}\Delta t\right)$
                $G_{i,j,k}^{\eta,forc} = \delta_{k,1} F_{i,j} / \left(\rho_0 h_{i,j,k} \Delta z_k\right)$
                $G_{i,j,k}^{\eta,convH} = \left[\left(U_{i,j,k} - U_{i+1,j,k}\right)\Delta y_{i,j} + \left(V_{i,j,k} - V_{i,j+1,k}\right)\Delta x_{i,j}\right] / \left(A_{i,j} h_{i,j,k}\right)$
                $G_{i,j,k}^{\eta,convV} = \left[\left(1 - \delta_{k,K}\right) W_{i,j,k+1} - \left(1 - \delta_{k,1}\right) W_{i,j,k}\right] / \left(h_{i,j,k}\Delta z_k\right)$
                $G_{i,j,k}^{\eta,conv} = G_{i,j,k}^{\eta,convH} + G_{i,j,k}^{\eta,convV}$
            end for
        end for
    end for
end for
```

Discretizing the Primitive Equations: Volume



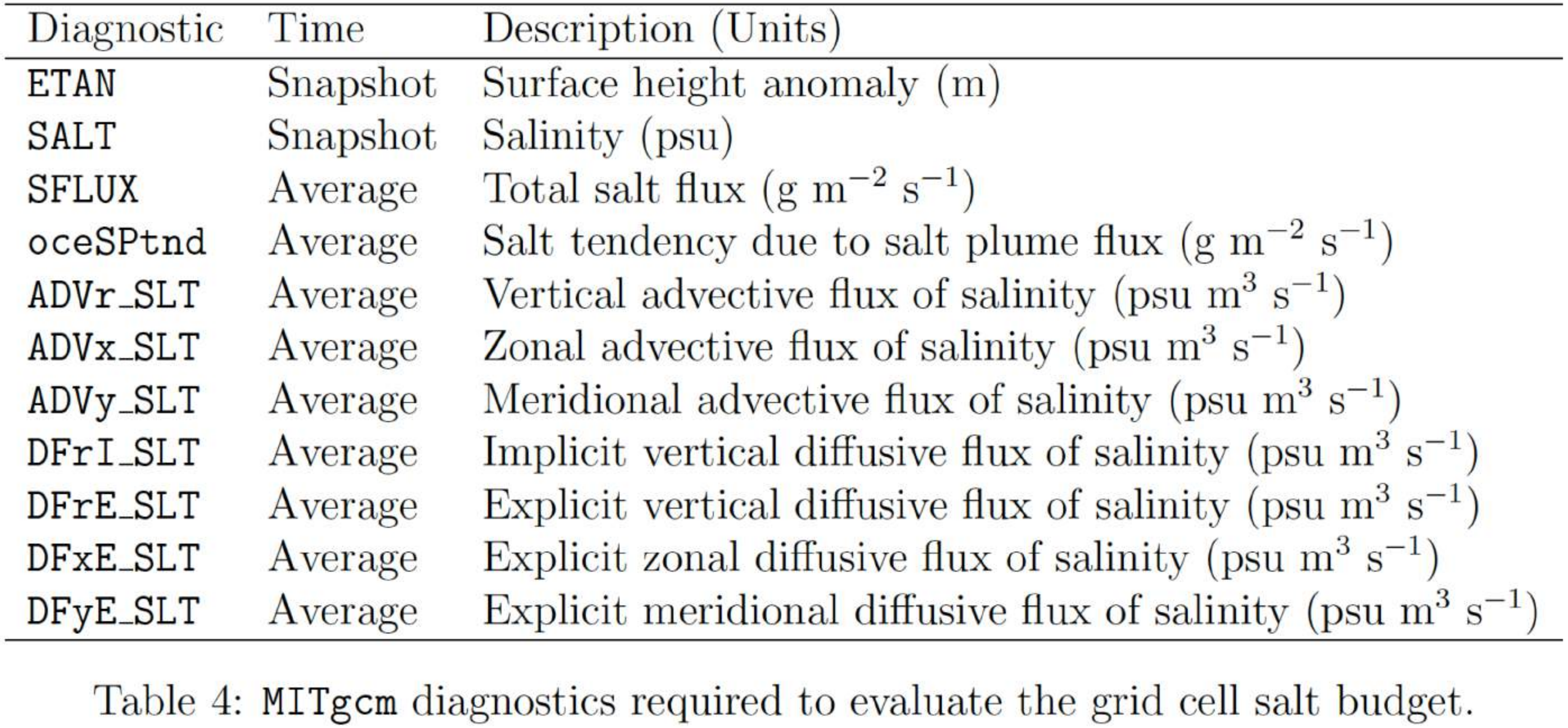
Discretizing the Primitive Equations: Salt

$$\underbrace{\frac{\partial\left(s^{*}S\right)}{\partial t}}_{G^{S,tot}} = \underbrace{-\nabla_{z^{*}}\left(s^{*}S\mathbf{v}_{res}\right) - \frac{\partial\left(Sw_{res}\right)}{\partial z^{*}}}_{G^{S,adv}} + \underbrace{s^{*}\mathcal{F}_{S}}_{G^{S,forc}} + \underbrace{s^{*}D_{S}}_{G^{S,diff}}$$

$$\frac{s^{*n+1}S^{n+3/2} - s^{*n}S^{n+1/2}}{\Delta t} = \mathcal{A}\left(S, \mathbf{u}^{n+1} + \mathbf{u}_{b}^{n+1}\right) + s^{*n}\left(\mathcal{F}_{S}^{n+1} + D_{\sigma,S}^{n+1/2} + D_{\perp,S}^{n+3/2}\right)$$

Forget et al. (2015), Geosci. Mod. Dev.

Discretizing the Primitive Equations: Salt



Diagnostic	Time	Description (Units)
`ETAN`	Snapshot	Surface height anomaly (m)
`SALT`	Snapshot	Salinity (psu)
`SFLUX`	Average	Total salt flux (g m^{-2} s^{-1})
`oceSPtnd`	Average	Salt tendency due to salt plume flux (g m^{-2} s^{-1})
`ADVr_SLT`	Average	Vertical advective flux of salinity (psu m^3 s^{-1})
`ADVx_SLT`	Average	Zonal advective flux of salinity (psu m^3 s^{-1})
`ADVy_SLT`	Average	Meridional advective flux of salinity (psu m^3 s^{-1})
`DFrI_SLT`	Average	Implicit vertical diffusive flux of salinity (psu m^3 s^{-1})
`DFrE_SLT`	Average	Explicit vertical diffusive flux of salinity (psu m^3 s^{-1})
`DFxE_SLT`	Average	Explicit zonal diffusive flux of salinity (psu m^3 s^{-1})
`DFyE_SLT`	Average	Explicit meridional diffusive flux of salinity (psu m^3 s^{-1})

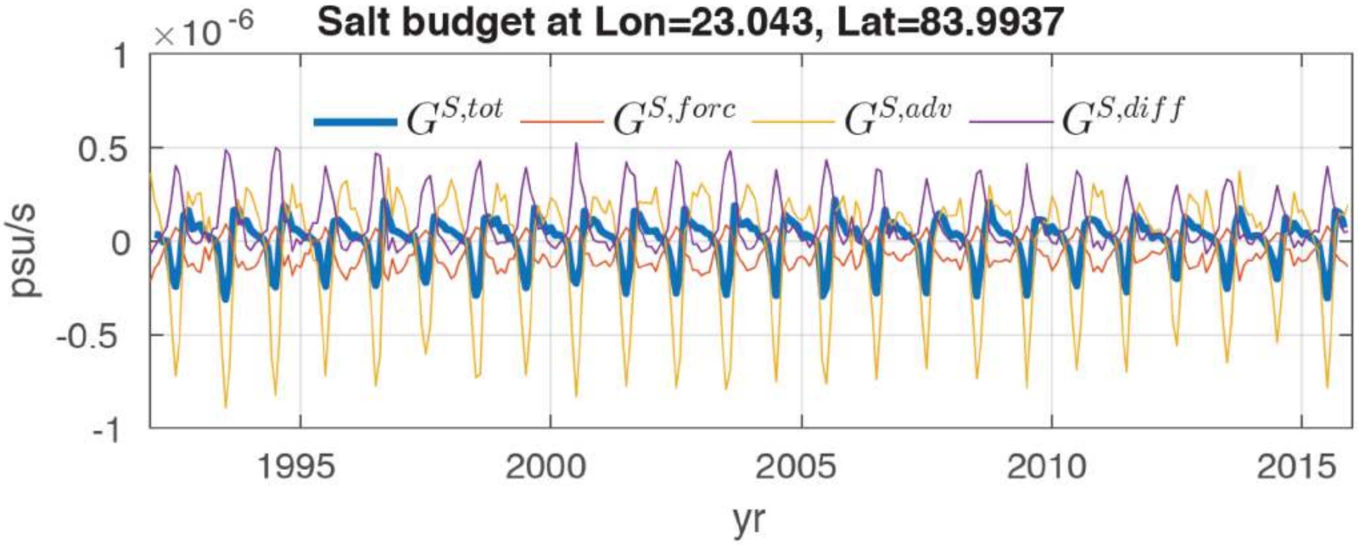
Table 4: `MITgcm` diagnostics required to evaluate the grid cell salt budget.

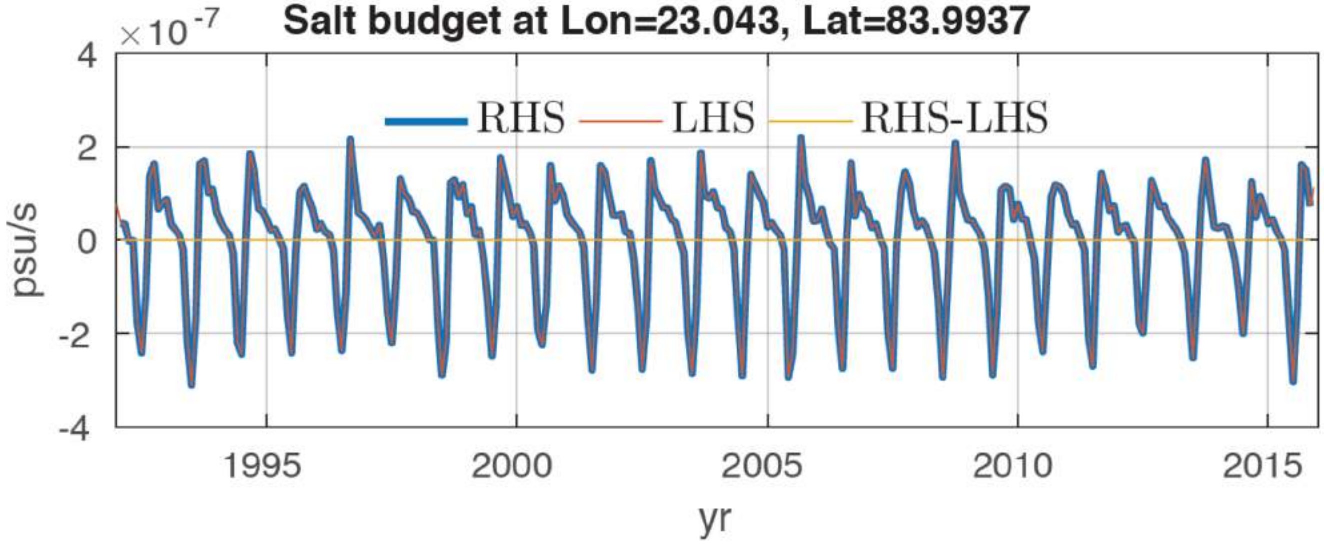
Discretizing the Primitive Equations: Salt

1.

```
for $t = t_1, t_2, \dots t_{T-1}, t_T$ do                              ▷ Loop over $T$ time steps (months) $t$
    $Q_{i,j} = \texttt{SFLUX}\{t\}$                                    ▷ 2-D **average** total surface salt flux over month $t$
    $P_{i,j,k} = \texttt{oceSPtnd}\{t\}$                               ▷ 3-D **average** salt plume tendency over month $t$
    $\rho_0 = 1029$                                                    ▷ Reference density (kg $m^{-3}$)
    for $i = i_1, i_2, \dots i_{I-1}, i_I$ do                          ▷ Loop over $I$ longitude cells $i$
        for $j = j_1, j_2, \dots j_{J-1}, j_J$ do                      ▷ Loop over $J$ latitude cells $j$
            for $k = k_1, k_2, \dots k_{K-1}, k_K$ do                  ▷ Loop over $K$ vertical cells $k$
                $G^{S,forc}_{i,j,k} = 0$
                if k=1 then
                    $G^{S,forc}_{i,j,k} = G^{S,forc}_{i,j,k} + Q_{i,j} / \left(\rho_0 h_{i,j,k} \Delta z_k\right)$
                end if
                $G^{S,forc}_{i,j,k} = G^{S,forc}_{i,j,k} + P_{i,j,k} / \left(\rho_0 h_{i,j,k} \Delta z_k\right)$
            end for
        end for
    end for
end for
```

Discretizing the Primitive Equations: Salt


Salt budget at Lon=23.043, Lat=83.9937


Salt budget at Lon=23.043, Lat=83.9937

Diagnosing Approximate Budgets: Salinity

$$\frac{\partial \left(s^* S\right)}{\partial t} = s^* \frac{\partial S}{\partial t} + S \frac{\partial s^*}{\partial t}$$

*** use chain rule**
*** combine salt & volume conservation equations**

$$\frac{\partial S}{\partial t} = -\frac{1}{s^*}\left[S\frac{\partial s^*}{\partial t} + \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) + \frac{\partial \left(S w_{res}\right)}{\partial z^*}\right] + \mathcal{F}_S + D_S$$

$$\underbrace{\frac{\partial S}{\partial t}}_{G^{\dagger,tot}} = \underbrace{\frac{1}{s^*}\left[S\nabla_{z^*}\left(s^* \mathbf{v}\right) + S\frac{\partial w}{\partial z^*} - \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) - \frac{\partial \left(S w_{res}\right)}{\partial z^*}\right]}_{G^{\dagger,adv}} + \underbrace{\mathcal{F}_S - S\mathcal{F}}_{G^{\dagger,forc}} + \underbrace{D_S}_{G^{\dagger,diff}}$$

Diagnosing Approximate Budgets: Salinity

Proportional to tendency in salt cons. eq.

$$\frac{\partial\left(s^* S\right)}{\partial t} = s^* \frac{\partial S}{\partial t} + S \frac{\partial s^*}{\partial t}$$

*** use chain rule**
*** combine salt & volume conservation equations**

$$\frac{\partial S}{\partial t} = -\frac{1}{s^*}\left[S \frac{\partial s^*}{\partial t} + \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) + \frac{\partial\left(S w_{res}\right)}{\partial z^*}\right] + \mathcal{F}_S + D_S$$

$$\underbrace{\frac{\partial S}{\partial t}}_{G^{\dagger,tot}} = \underbrace{\frac{1}{s^*}\left[S \nabla_{z^*}\left(s^* \mathbf{v}\right) + S \frac{\partial w}{\partial z^*} - \nabla_{z^*}\left(s^* S \mathbf{v}_{res}\right) - \frac{\partial\left(S w_{res}\right)}{\partial z^*}\right]}_{G^{\dagger,adv}} + \underbrace{\mathcal{F}_S - S\mathcal{F}}_{G^{\dagger,forc}} + \underbrace{D_S}_{G^{\dagger,diff}}$$

Diagnosing Approximate Budgets: Salinity

$$\frac{\partial\left(s^{*}S\right)}{\partial t}=s^{*}\frac{\partial S}{\partial t}+S\frac{\partial s^{*}}{\partial t}$$

*** use chain rule**
*** combine salt & volume conservation equations**

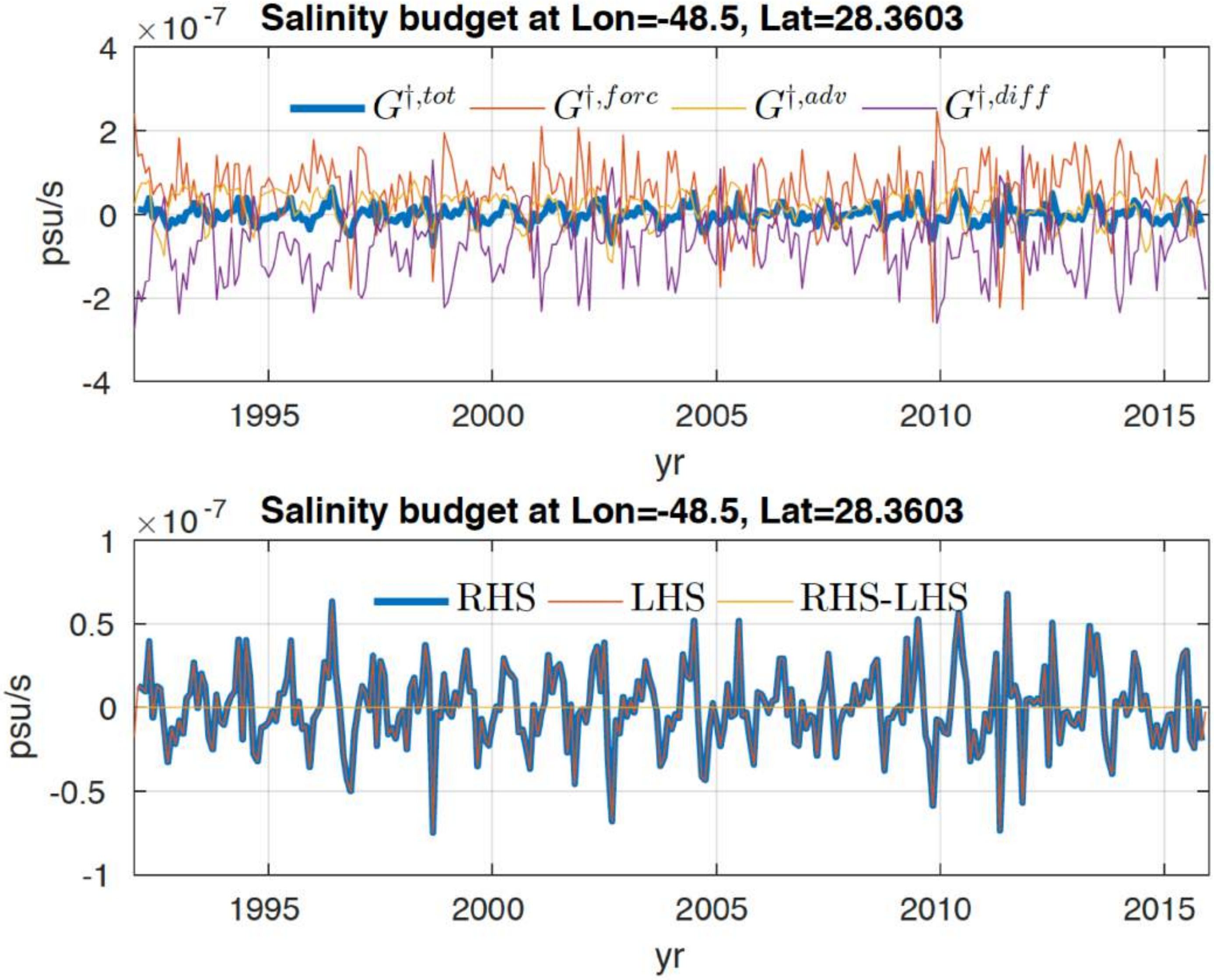
Proportional to tendency in salt cons. eq.

$$\frac{\partial S}{\partial t}=-\frac{1}{s^{*}}\left[S\frac{\partial s^{*}}{\partial t}+\nabla_{z^{*}}\left(s^{*}S\mathbf{v}_{res}\right)+\frac{\partial\left(Sw_{res}\right)}{\partial z^{*}}\right]+\mathcal{F}_{S}+D_{S}$$

$$\underbrace{\frac{\partial S}{\partial t}}_{G^{\dagger,tot}}=\underbrace{\frac{1}{s^{*}}\left[S\nabla_{z^{*}}\left(s^{*}\mathbf{v}\right)+S\frac{\partial w}{\partial z^{*}}-\nabla_{z^{*}}\left(s^{*}S\mathbf{v}_{res}\right)-\frac{\partial\left(Sw_{res}\right)}{\partial z^{*}}\right]}_{G^{\dagger,adv}}+\underbrace{\mathcal{F}_{S}-S\mathcal{F}}_{G^{\dagger,forc}}+\underbrace{D_{S}}_{G^{\dagger,diff}}$$

P-E+R

Diagnosing Approximate Budgets: Salinity



Thank you.

https://www.metoffice.gov.uk/weather/learn-about/weather/oceans/amoc