# Introduction to sea ice modelling

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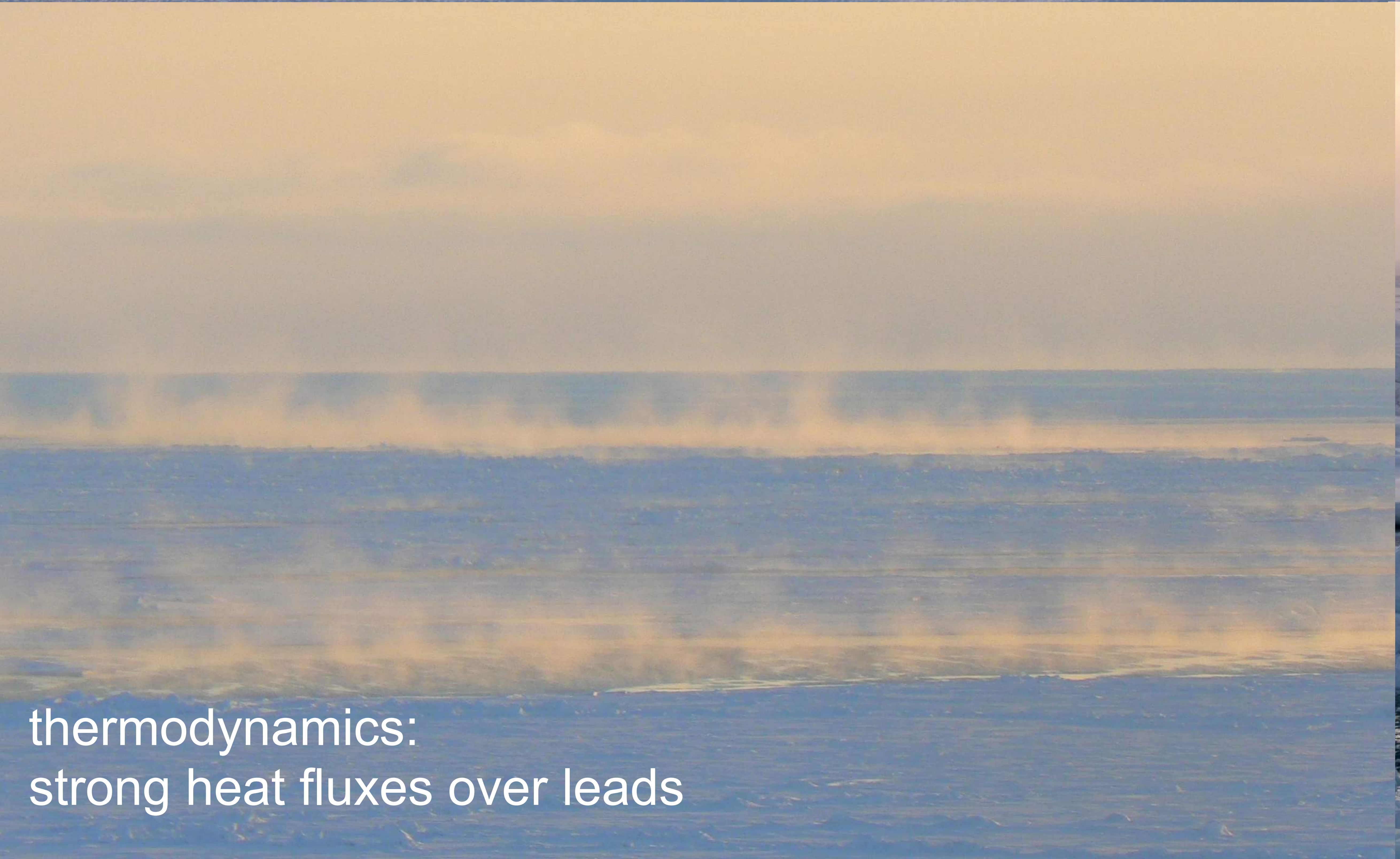


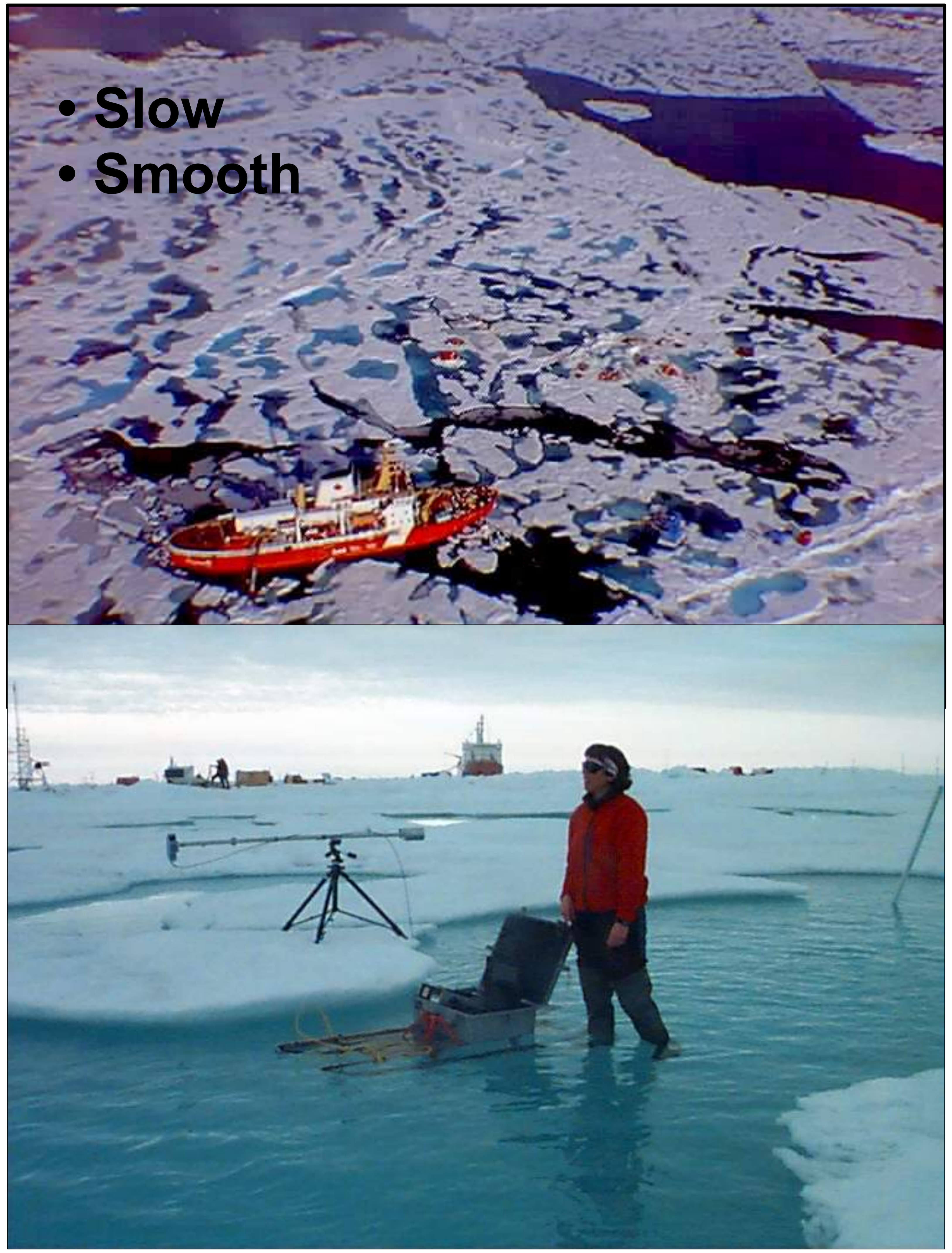
thermodynamics:
strong heat fluxes over leads

dynamics: ridges and leads, rubble, pack

Photos by M. Hoppmann, S. Hendricks, Ch. Lüpkes

# "Dynamic" duo for Sea Ice


*Dynamics*




*Thermodynamics*

Perovich, 2012, FAMOS

# Overview

- **(very short) Introduction:** Sea ice in the climate system

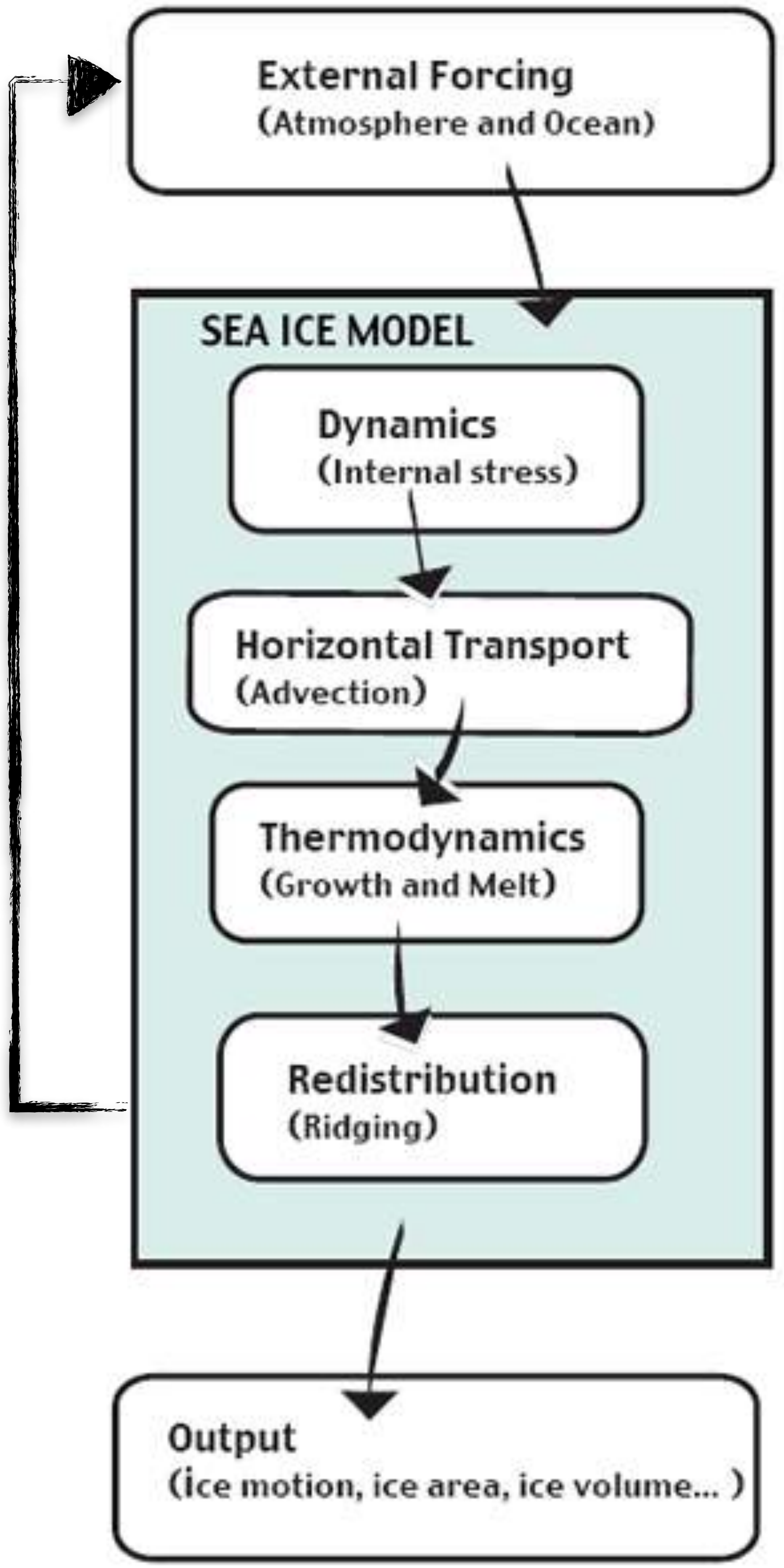
- **Thermodynamics**
  - heat balance
  - heat capacity
  - zero-layer, multi-layer models
  - salinity, brine, enthalpy
  - Snow on ice
  - advection
  - ice thickness distribution

- **Dynamics**
  - continuum assumption
  - momentum equations
  - surface stress
  - divergence of internal stress
  - rheology, isotropy, anisotropy, Viscous-Plastic, Maxwell-Elasto-Brittle, Mohr-Coulomb
  - ice thickness distribution and ridging

- **Numerical models**
  - solution techniques specific to sea-ice models
  - implicit solvers
    - Picard/fixed point/FGMRES, JFNK
  - explicit solvers:
    - EVP, mEVP, aEVP
    - EAP
  - new rheologies
  - Discrete Element Models

- **Sea ice models in ECCO -> Ian's talk**
- **Biogeochemistry in sea ice models**

# Sea ice model equations




Flowchart stolen from
https://nsidc.org/cryosphere/seaice/study/modeling.html

# thermodynamics: heat balance

- ice-enthalpy includes heat and chemical potential (salinity)

$$\rho \frac{\partial}{\partial t} E = \frac{\partial}{\partial z}\left(\kappa \frac{\partial T}{\partial t}\right) + Q, \qquad E(S,T) = c_{p,i}(T + \mu S) - \Lambda\left(1 - \phi\right) - c_{p,w}\mu S$$

brine fraction

- often simply:

$$E = c_p T$$

- with boundary conditions

top:

$$\kappa \frac{\partial T}{\partial z} = Q_T \downarrow$$

bottom:

$$T = T_f = T_f(S)$$

$$\rho \Lambda \frac{\partial H}{\partial t} = -\kappa \frac{\partial T}{\partial z}\bigg|_{\text{bottom}} - Q_w \uparrow$$

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# Stefan's law of ice growth (following Leppäranta, 1993)

- assumptions:
  - no thermal inertial: $E = 0$
  - no internal heat source: $Q = 0$
  - no heat flux from ocean: $Q_w = 0$
  - known surface temperature $T_0 = T_0(t)$

➡ constant temperature profiles (0-layer mode):

$$\frac{\partial T}{\partial z} = \text{constant}$$

$$\rho\Lambda \frac{dH}{dt} = -\kappa \frac{\partial T}{\partial z} = \kappa \frac{T_f - T_0(t)}{H}$$

$$H\frac{dH}{dt} = \frac{1}{2}\frac{dH^2}{dt} dt = \frac{\kappa}{\rho\Lambda}\left[T_f - T_0(t)\right]$$

$$\int_0^t \frac{dH^2}{dt} dt' = H(t)^2 - H_0^2 = \frac{2\kappa}{\rho\Lambda}\int_0^t \left[T_f - T_0(t')\right] dt'$$

sum of negative degree days

$$H(t) = \sqrt{H_0^2 + \frac{2\kappa}{\rho\Lambda}\int_0^t \left[T_f - T_0(t')\right] dt'}$$

gives typically 140cm for 180 days of -10K freezing conditions

# Sea ice model equations: "0-layer thermodynamics"

(Semptner, 1976, 1984)

$$\rho\frac{\partial E}{\partial t} = \rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial t}\right) + Q$$

- no internal heat source: $Q = 0$
- no thermal inertia => instantaneous temperature adjustment

$$c_p = 0 \Rightarrow \frac{\partial T}{\partial z} = \text{constant} = \frac{T_0 - T_f}{h}$$

- only surface boundary conditions remain: $\frac{\kappa}{h}(T_0 - T_f) = Q_T \downarrow$
- with $Q_T \downarrow \; = Q_{SW\downarrow}(1-\alpha) + \epsilon Q_{LW\downarrow} - Q_{LW\uparrow}(T_0) + Q_{LH}(T_0) + Q_{SH}(T_0)$

$$\Rightarrow T_0 \Rightarrow Q_T(T_0) \Rightarrow \rho\Lambda\frac{\partial H}{\partial t} = \kappa\frac{T_f - T_0}{H} - Q_w$$

# Sea ice albedo for $\mathbf{Q_{SW\downarrow}(1-\alpha)}$

- simple parameterisations with albedo for ice and snow in freezing (brighter) or melting (darker) conditions
- use melt-pond physics to estimate albedo (e.g., Taylor and Feltham, 2004, Flocco and Feltham, 2007)
- effects of ageing snow and ice, multiple-scattering, absorptive effects of inclusions such as dust and algae (biogeochemistry!)

- important tuning parameter

# Snow on sea ice

- snow is a very good insulator:
  - changes surface albedo
  - usually limits shortwave penetration
  - changes vertical diffusion of temperature (from continuity at interface, e.g. Leppäranta, 1991) in 0-layer model:

$$\kappa_{eff} = \frac{\kappa_{ice}\,\kappa_{snow}}{\kappa_{snow}\,h_{ice} + \kappa_{ice}\,h_{snow}}$$

- snow-ice: refreezing of flooded snow on sea ice, especially in Antarctica

# going back to more general thermodynamics

- ice-enthalpy

$$\rho\frac{\partial}{\partial t}E = \frac{\partial}{\partial z}\left(\kappa\frac{\partial T}{\partial t}\right) + Q$$

- Simplest model is a three-layer model (two ice layers, one snow layer, e.g. Winton, 2000), more layers (Bitz and Lipscomb, 1999) used CICE/ICEPACK.
- any excess conductive heat flux ($k\partial T/\partial z$) through the ice leads to freezing or melting: change of volume $S_h\,(=dh/dt)$
- Hibler (1979): lateral freezing and melting

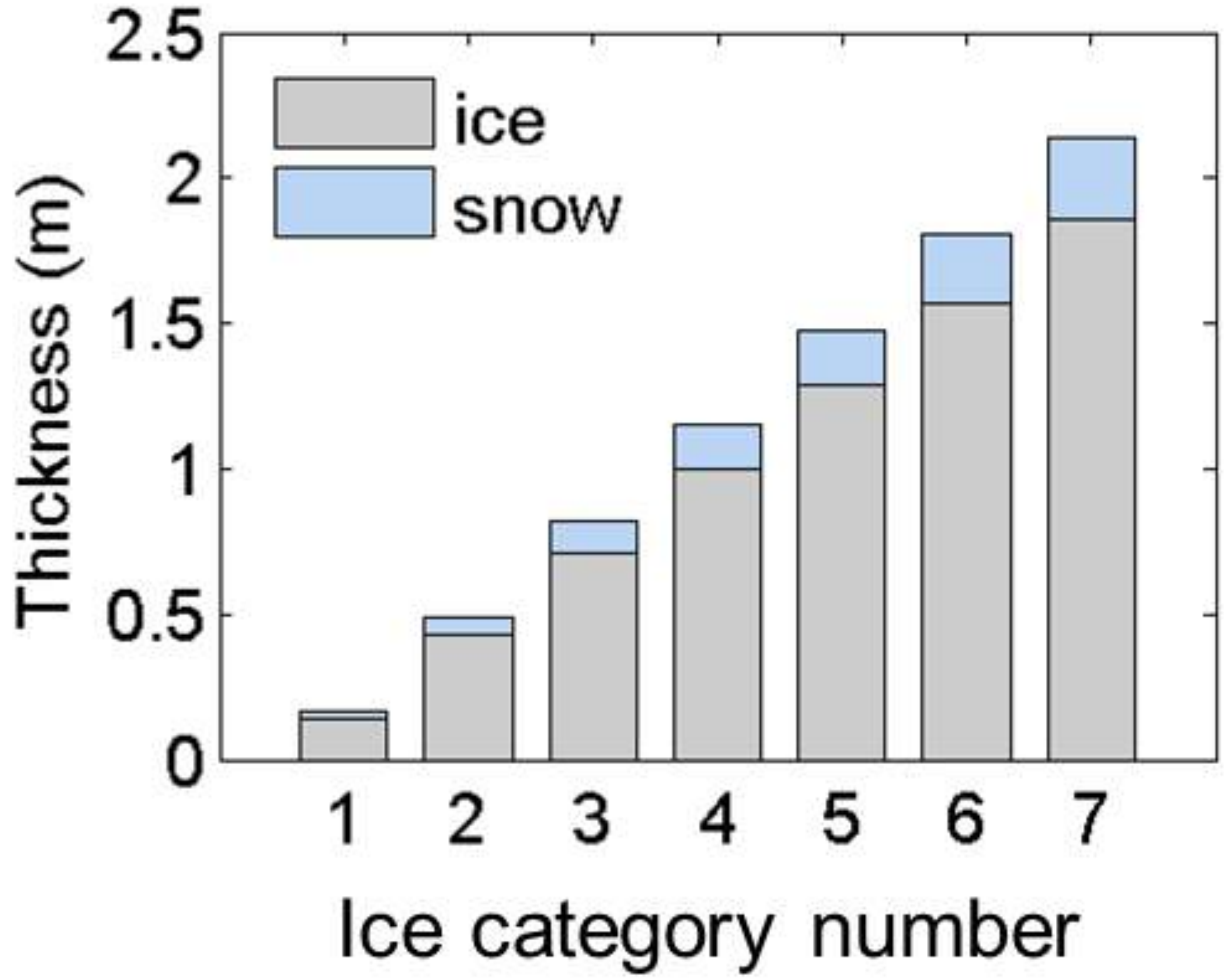
$$S_c = \frac{1-c}{h_0}\max\left(S_h, 0\right) + \frac{c}{2h}\min\left(S_h, 0\right)$$

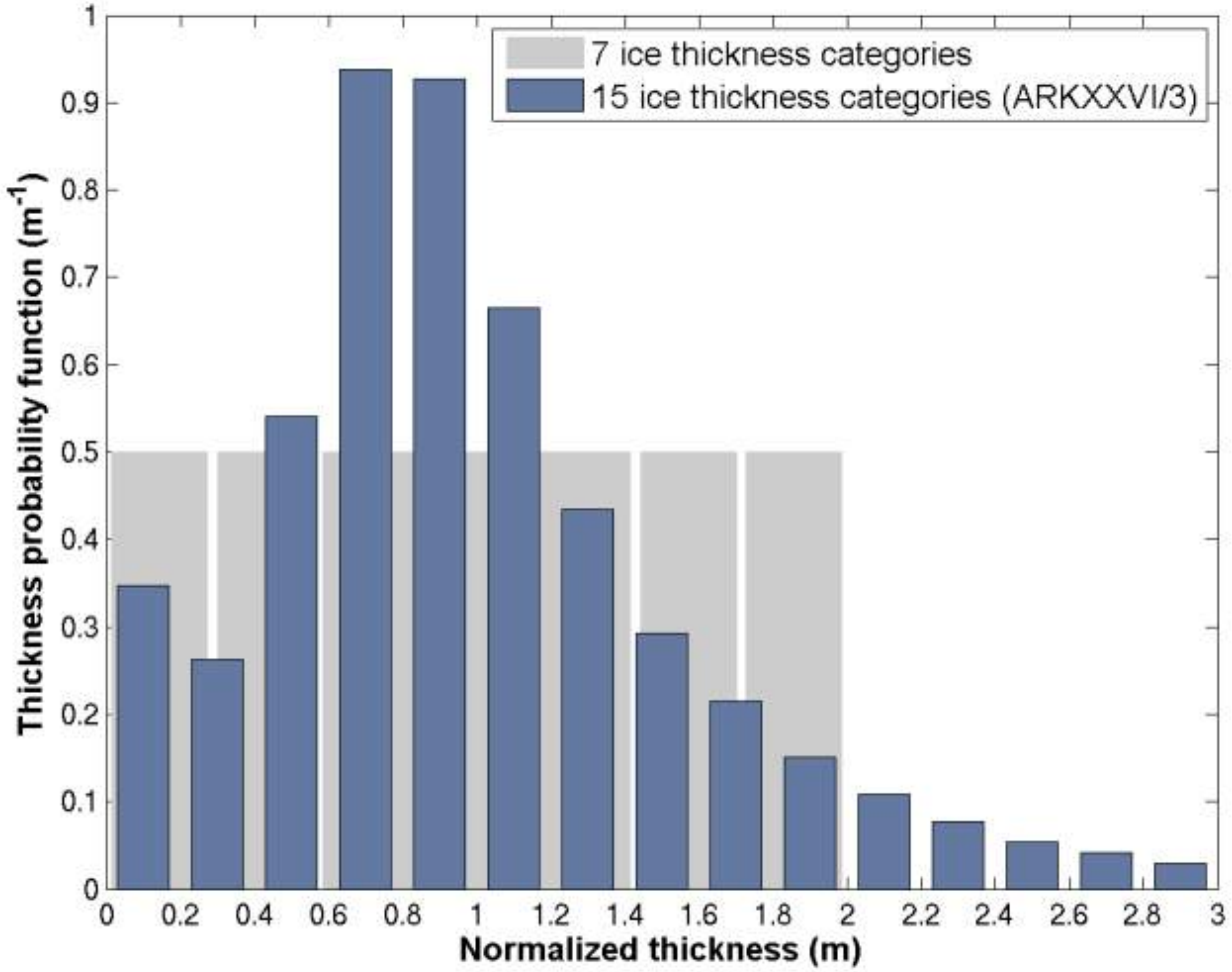
lead closing parameter

# ice thickness distribution (static)

- thick ice (especially with a snow layer) is a good insulator and limits new ice growth
- simple parameterization scales distribution by mean thickness to always allow thin ice

$$Q = (1-c)\,Q_0 + c\sum_{k=1}^{N} Q\left(\frac{2k-1}{N}\frac{h}{c}\right)$$



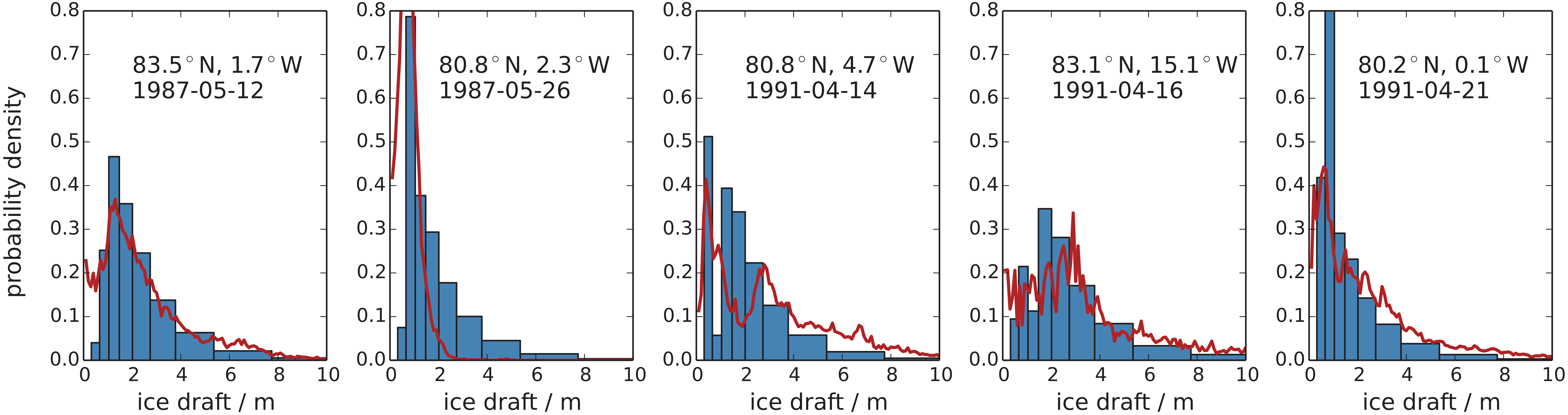

images: Castro-Morales et al. (2014)

# dynamic ice thickness distribution: redistribution + ridging

- ice concentration equation is replaced by an equation for thickness distribution function *g(h)*

ridging function

$$\frac{\partial c}{\partial t} = -\nabla \cdot (c\,\mathbf{u}) + S_c \longrightarrow \frac{\partial g}{\partial t} = -\nabla \cdot (\mathbf{u}g) - \frac{\partial}{\partial h}(fg) + \Psi$$




red: observations, blue: model

from Ungermann and Losch (2018)

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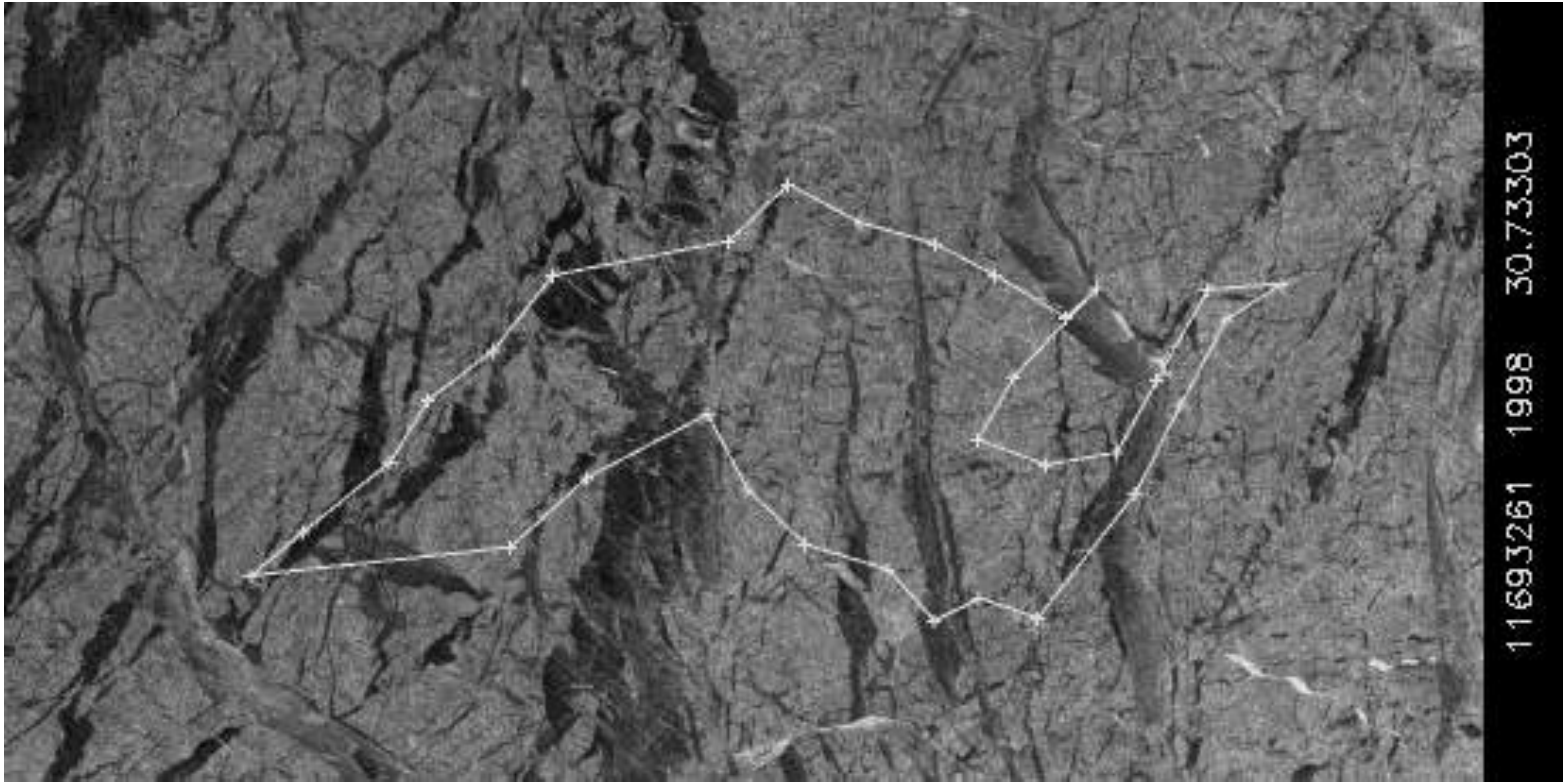
# Dynamics and Deformation

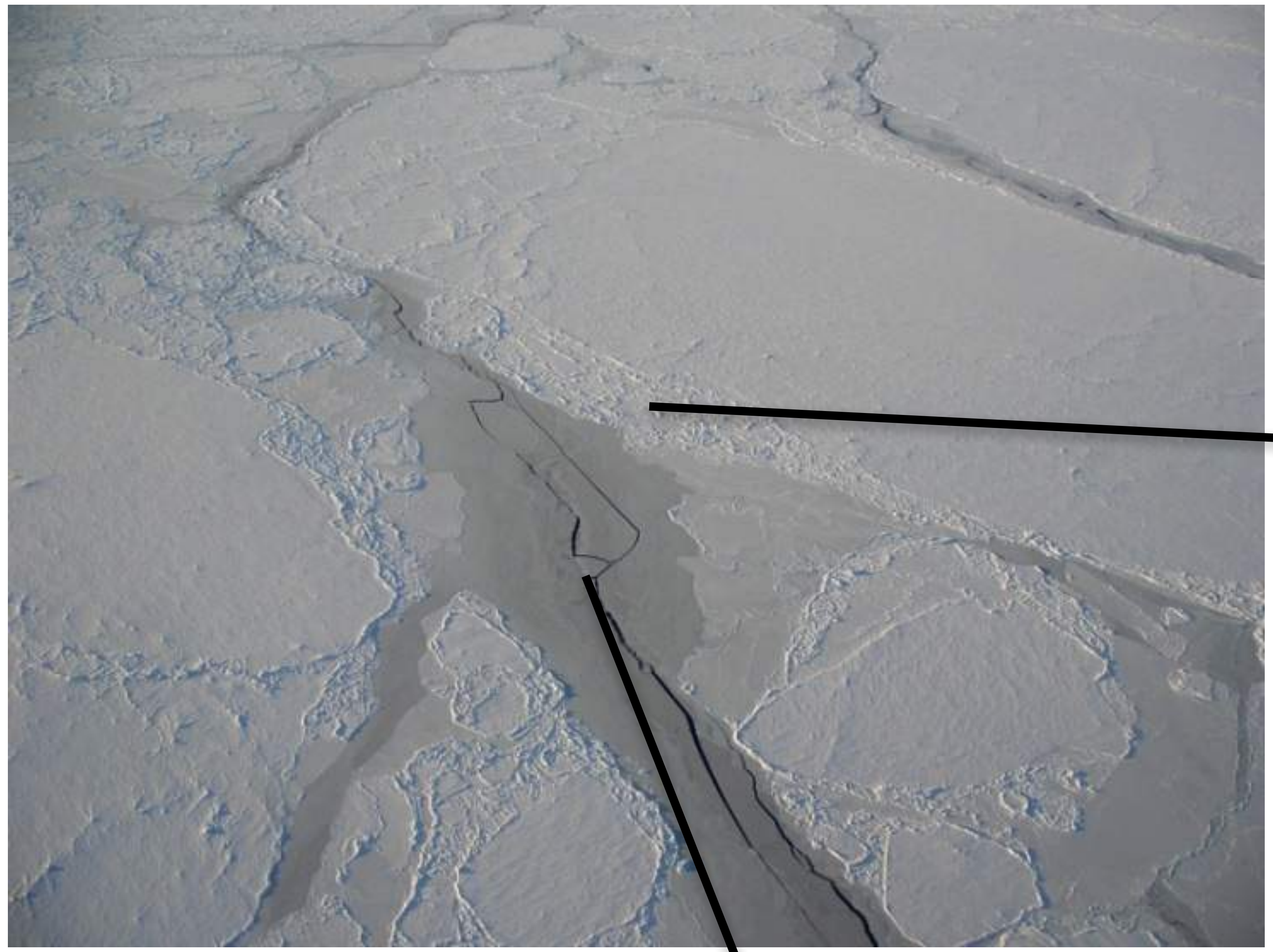





(RGPS data near SHEBA drift station, 1997, R. Kwok)

# Sea Ice Deformation



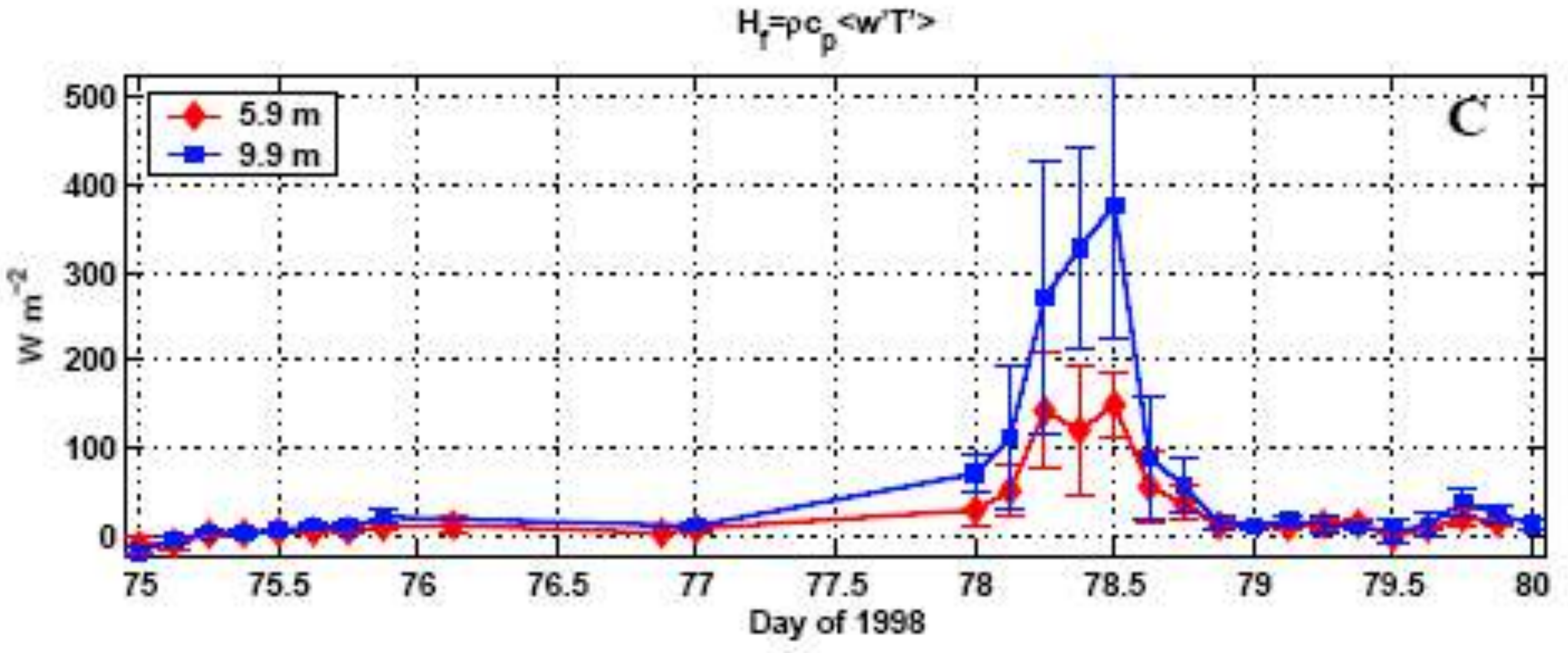


ice compression and shear:
ridges, rubble fields

breaking ice:
cracks, leads

# Importance of sea ice deformations

- Affect the thickness distribution through formation of ridges and leads.
- Heat flux through new leads is 1-2 orders of mag higher than over thick ice (Maykut, 1978)
- 25-40% of new ice formation occurs in leads (Kwok, 2006)
- Ridges affect the air-ice and ice-ocean drag
- Ocean upwelling associated to shear




McPhee et al., 2005

# Digression: Newtonian Fluid

1. Fluid is continuous + stresses are a linear function of the strain rates
2. Fluid is isotropic
3. In the limit where the strain rates go to zero, the stresses must reduce to the hydrostatic pressure

$$\rho \frac{d\mathbf{u}}{dt} = \nabla \cdot \sigma + R, \qquad R = \text{other terms}$$

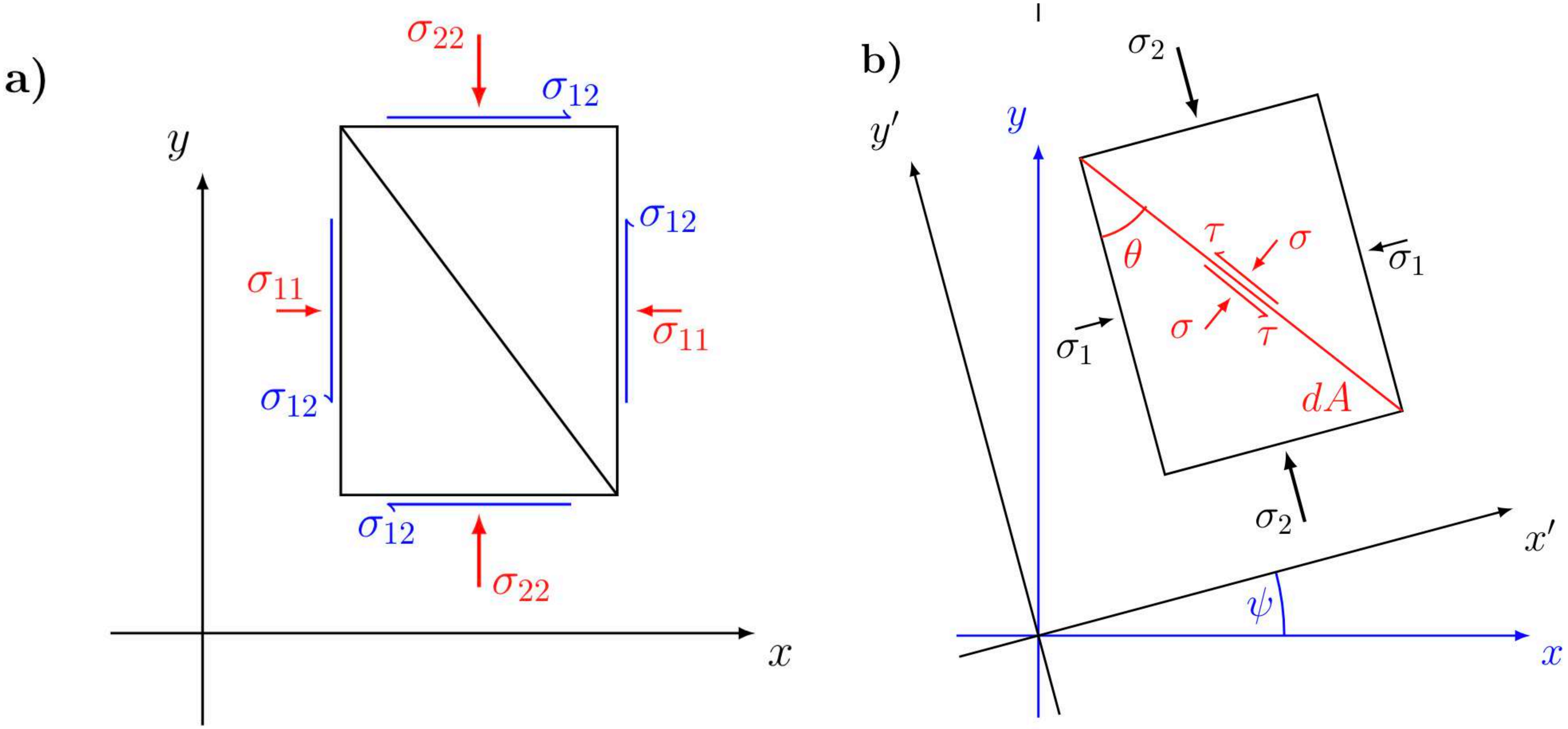
for PE (ocean): $\sigma = \nu \nabla \mathbf{u} - p \Rightarrow \nabla \cdot \sigma = \nabla \left( \nu \nabla \mathbf{u} - p \right)$

strain rates $(\nabla \mathbf{u})_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$

# Sea ice is different (AIDJEX model)

- ensemble of many ice floes with interactions
- continuity assumption at large scales (questionable)
- non-Newtonian (non-normal) fluid (honey vs. mayonnaise)
- strong in compression, weak in tension, intermediate in shear
- elastic response to small perturbations, plastic to large pert.
- => plastic-elastic model (Coon 1974)
- viscous-plastic framework is similar, but numerically simpler (Hibler 1977, 1979), small strain rates lead to viscous creep.
- granular material (like sand: floes = grains) => Mohr-Coulomb law of failure relates shear to normal stress: $\tau = \mu\sigma + c$

# Stress tensor




$$\sigma = \begin{pmatrix} \sigma_{11}\sigma_{21} \\ \sigma_{12}\sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_{xx}\sigma_{yx} \\ \sigma_{xy}\sigma_{yy} \end{pmatrix} \Rightarrow \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$

principle stresses

$$\sigma_{1,2} = \frac{\sigma_{11} + \sigma_{22}}{2} \pm \sqrt{\frac{(\sigma_{11} - \sigma_{22})^2}{4} + \sigma_{12}\sigma_{21}}$$

normal stress $\sigma$

$$\sigma_I = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{1}{2}(\sigma_{11} + \sigma_{22})$$

shear stress $\tau$

$$\sigma_{II} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}\sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}\sigma_{21}}$$

- assumption: symmetric $\Rightarrow \sigma_{21} = \sigma_{12}$
- Mohr-Coulomb law: $\tau = \mu\sigma + c$ or $\sigma_{II} = \mu\sigma_I + c/2$

# principal stress plane and yield curve, plastic limit

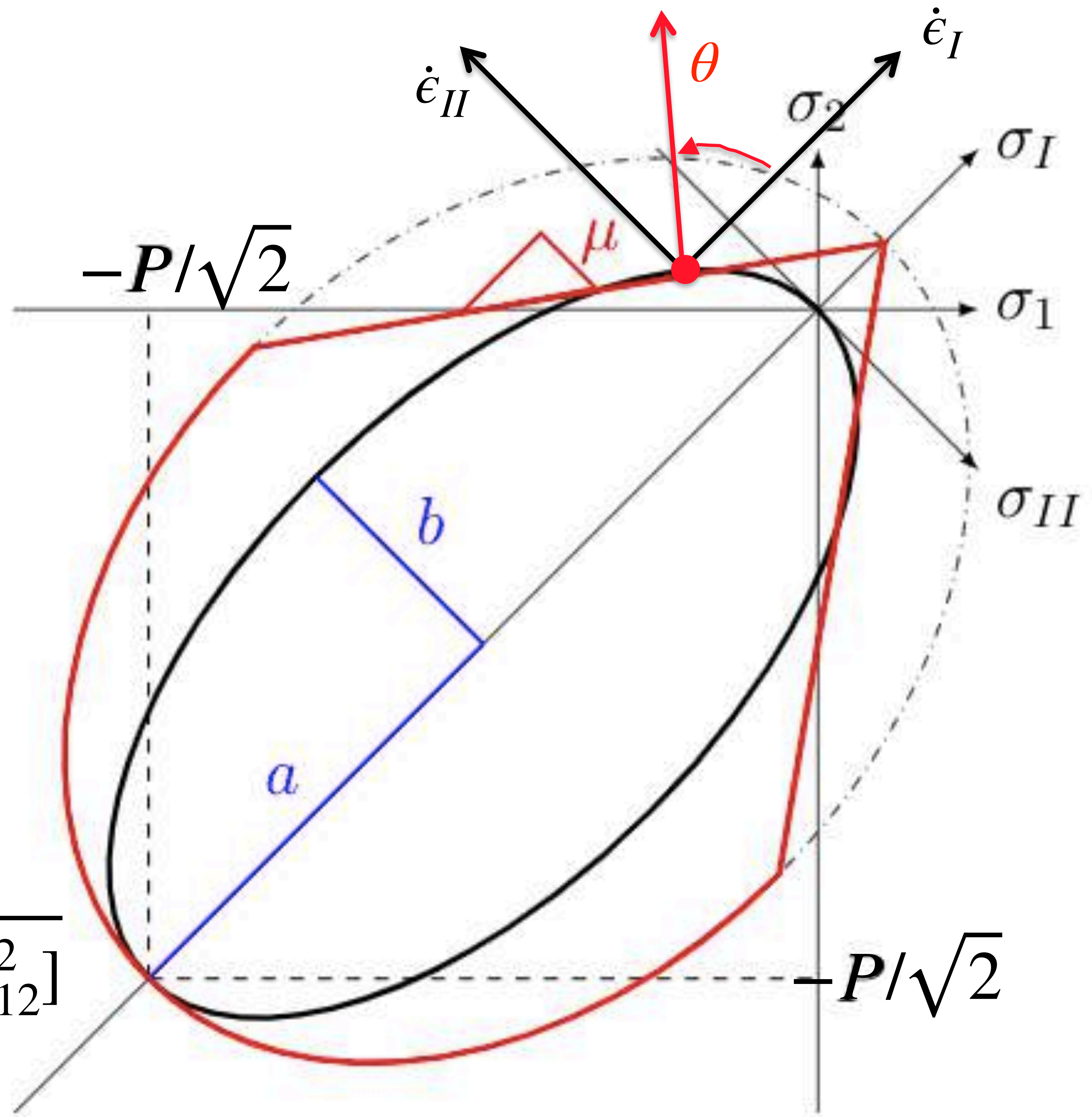
$$\sigma = \begin{pmatrix} \sigma_{11}\sigma_{21} \\ \sigma_{12}\sigma_{22} \end{pmatrix} \Rightarrow \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \Rightarrow \sigma_I \text{ and } \sigma_{II}$$

elliptical yield curve: $F = \left(\frac{\sigma_I + P/2}{P/2}\right)^2 + \left(\frac{\sigma_{II}}{P/(2e)}\right)^2 - 1 = 0$

with normal flow rule: $\dot{\epsilon}_{ij} = \lambda \frac{\partial F(\sigma)}{\partial \sigma_{ij}} \quad \left( = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$

$\Rightarrow$ constitutive relation: $\sigma_{ij} = 2\eta\,\dot{\epsilon}_{ij} + \delta_{ij}\left(\left[\zeta - \eta\right]\dot{\epsilon}_{kk} - \frac{P}{2}\right)$

$$e = \frac{a}{b}, \quad \zeta = \frac{P}{2\Delta}, \quad \eta = \frac{\zeta}{e^2}, \quad \Delta = \sqrt{(\dot{\epsilon}_{11} + \dot{\epsilon}_{22})^2 + e^{-2}[(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2]}$$



# consequence: sea ice dynamics are very non-linear

$$m\frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \sigma + R, \qquad R = \text{other terms}$$

with $\sigma_{ij} = \frac{P}{2\Delta}\left\{2\dot\epsilon_{ij}\,e^{-2} + \left[(1-e^{-2})(\dot\epsilon_{11}+\dot\epsilon_{22}) - \Delta\right]\delta_{ij}\right\}$

with abbreviations

$$\Delta = \sqrt{(\dot\epsilon_{11}+\dot\epsilon_{22})^2 + e^{-2}\left[(\dot\epsilon_{11}-\dot\epsilon_{22})^2 + 4\dot\epsilon_{12}\right]}$$

$$\dot\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \text{(strain rates)}$$

$$\longrightarrow \quad m\frac{\partial \mathbf{u}}{\partial t} \propto \frac{\partial}{\partial x_i}\left(\frac{P}{\Delta}\frac{\partial u_i}{\partial x_j}\right) + \text{similar terms}$$

ice strength parametrizations:

Hibler (1979): $P = P^\star h\, e^{-C^\star(1-c)}$

Rothrock (1975): $P = C_f\, C_p \int_0^\infty h^2 \omega_r(h)\, dh$

# Sea ice dynamics and solvers

- Picard solvers (LSR, Krylov)
- JFNK solver
- EVP solvers: mEVP, aEVP

- new MEB rheology

- discrete element models (DEM)

# solution techniques: Picard method

$$\mathbf{A}(\mathbf{u}) \cdot \mathbf{u} = \mathbf{b}$$

$$\Rightarrow \text{solve} \qquad \mathbf{A}(\mathbf{u}_{n-1}) \cdot \mathbf{u}_n = \mathbf{b}$$

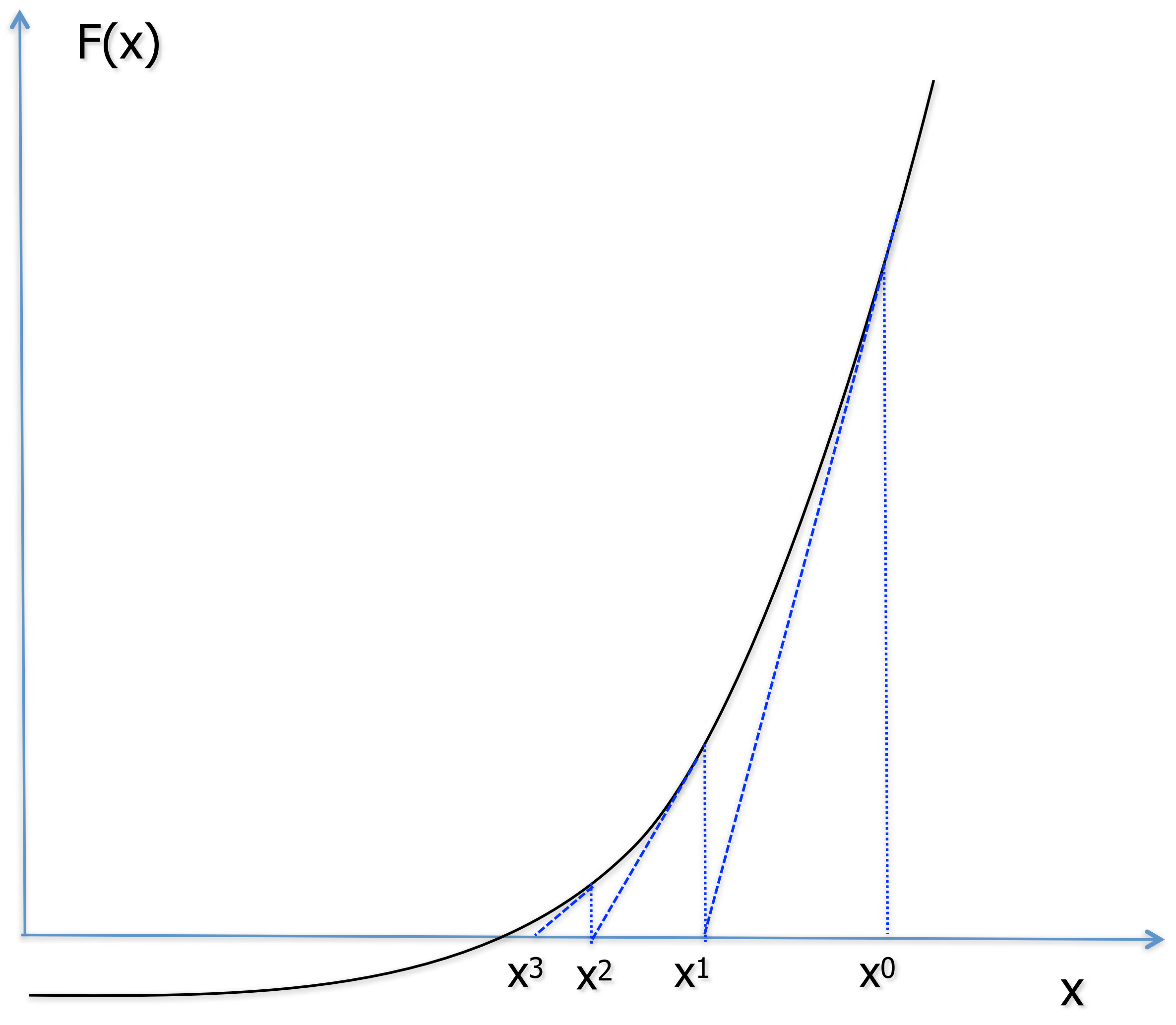
- traditional method, e.g., PSOR, Hibler (1979), LSOR, Zhang and Hibler (1997), (Gauss-Seidel) for linear solver
- Krylov method for linear solver (Lemieux and Tremblay, 2009), requires preconditioner
- stable, but slow

# solution techniques: JFNK solver

$$\mathbf{F}(\mathbf{u}) = \mathbf{A}(\mathbf{u}) \cdot \mathbf{u} - \mathbf{b}$$

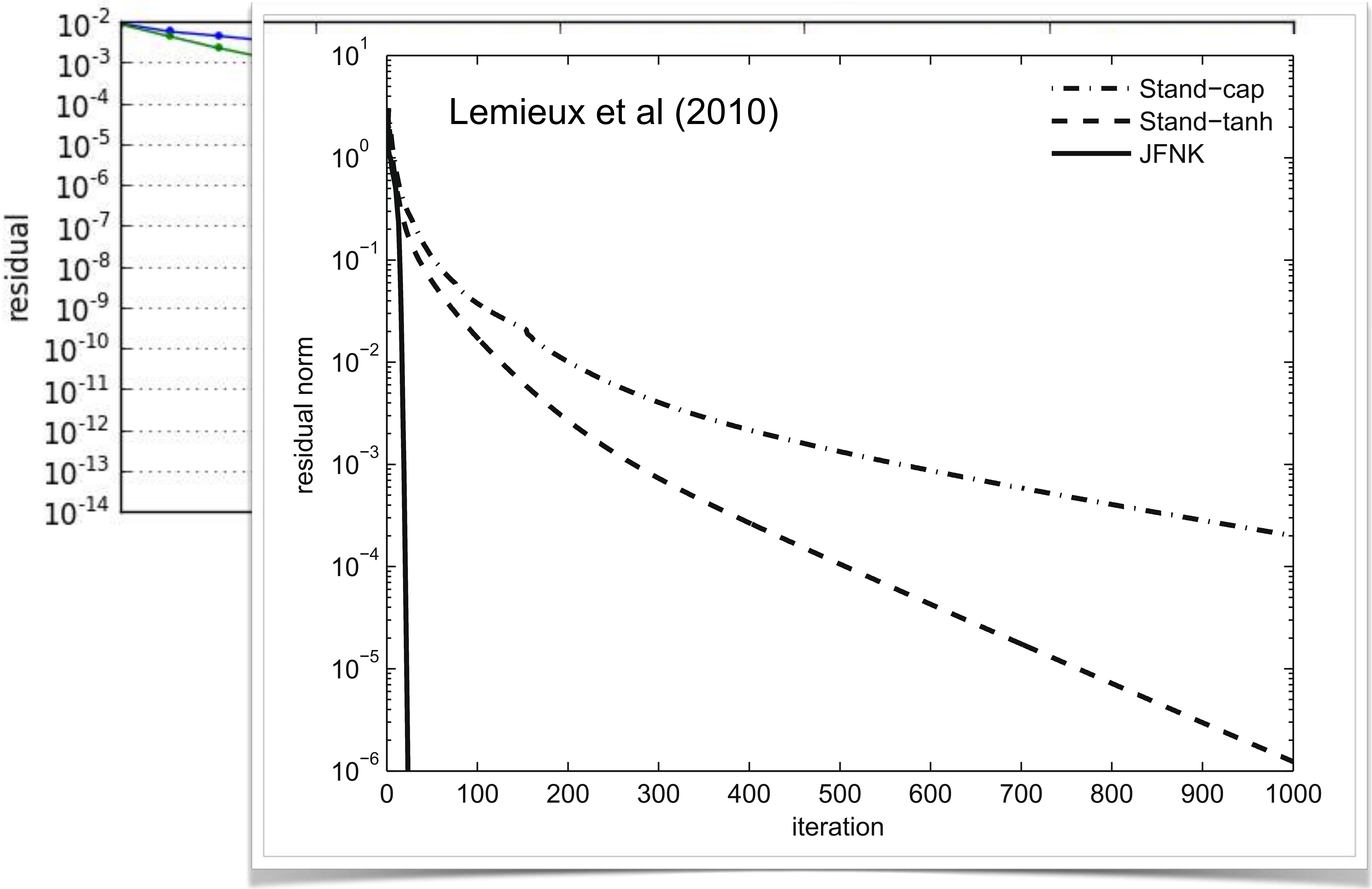
$$\mathbf{F}(\mathbf{u}_n) = \mathbf{F}(\mathbf{u}_{n-1}) + \mathbf{F}'\Big|_{\mathbf{u}_{n-1}} \delta\mathbf{u} \overset{!}{=} 0$$

$\Rightarrow$ solve $\quad \mathbf{F}'_{n-1}\,\delta\mathbf{u} = -\,\mathbf{F}(\mathbf{u}_{n-1}) \quad \Rightarrow \quad \mathbf{u}_n = \mathbf{u}_{n-1} + \delta\mathbf{u}$

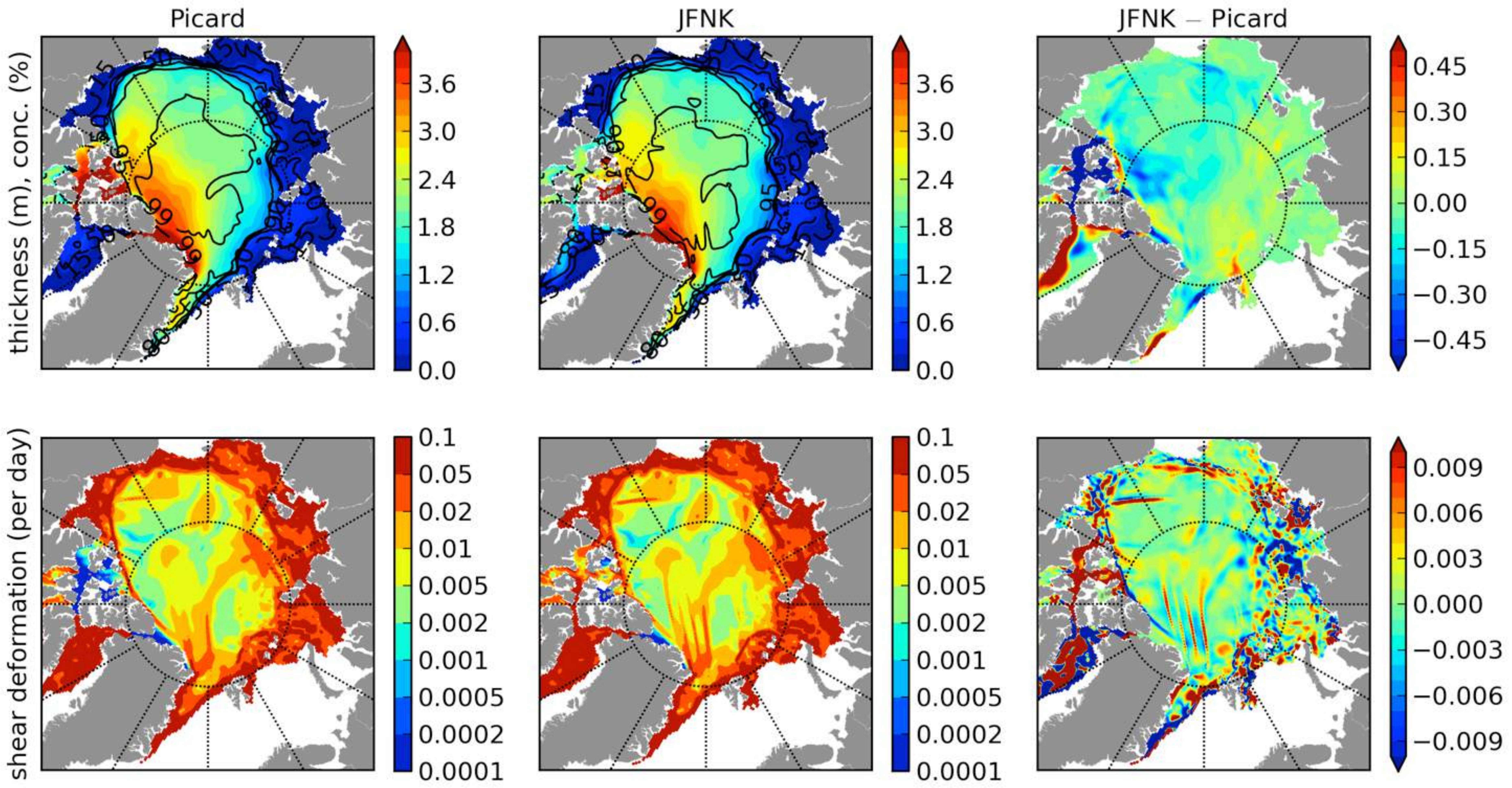



- better (quadratic) convergence near minimum (Lemieux et al. 2010, 2012, Losch et al 2014)
- preconditioner for Krylov solver necessary
- expensive
- unstable, especially at high resolution
- stabilization (e.g. Mehlmann and Richter 2017, involves mixing JFNK and Picard methods)

# Picard vs. JFNK



# Does it matter?




Losch et al. (2014)

# solution method: EVP variants

$$\sigma_{ij} = \frac{P}{2\Delta}\left\{2\dot{\epsilon}_{ij}e^{-2} + \left[(1-e^{-2})(\dot{\epsilon}_{11}+\dot{\epsilon}_{22}) - \Delta\right]\delta_{ij}\right\}$$

$$\Leftrightarrow \qquad \frac{\Delta e^2}{P}\sigma_{ij} + \left[\frac{\Delta(1-e^2)}{2P}(\sigma_{11}+\sigma_{22}) + \frac{\Delta}{2}\right]\delta_{ij} = \dot{\epsilon}_{ij}$$

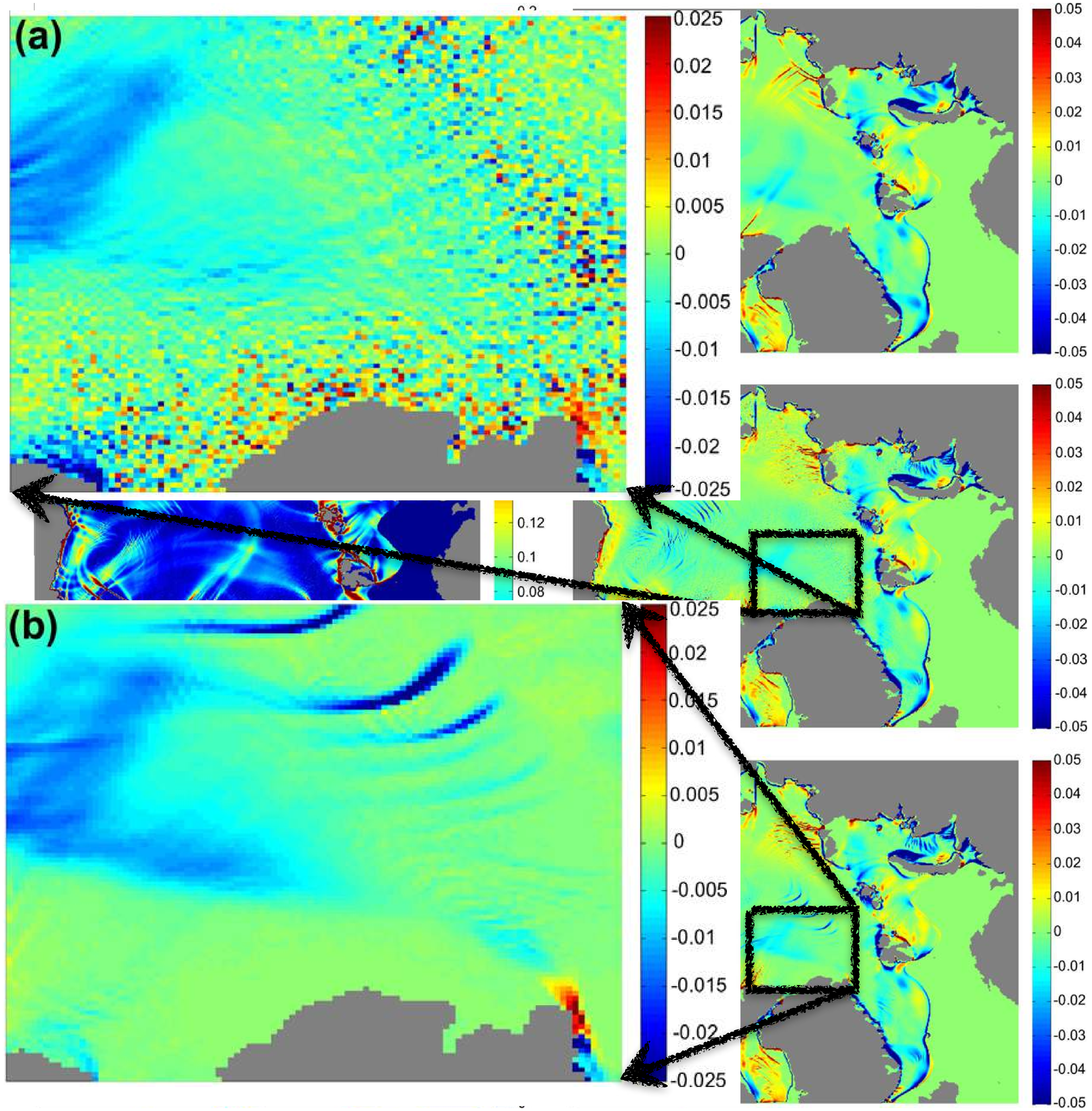
- Hunke and Dukowicz (1997)
- does not converge (definitely not to VP, Lemieux et al. 2012, Losch and Danilov 2012)
- adding inertial term to momentum equations fixes convergence (Lemieux et al. 2012, Bouillon et al 2013)
- m(odified)EVP, a(daptive)EVP (Kimmritz et al 2015, 2016, 2017)

# issues with conventional EVP

reference

EVP, 120 sub-cycles

EVP, 1980 sub-cycles




Lemieux et al. (2012), shear and divergence (per day)

# New EVP equations

based on Lemieux et al. (2012), Bouillon et al. (2013), add “inertial-like” term to momentum equations

$$\boldsymbol{\sigma}^{p+1} - \boldsymbol{\sigma}^{p} = \frac{1}{\alpha}\Big(\boldsymbol{\sigma}(\mathbf{u}^{p}) - \boldsymbol{\sigma}^{p}\Big),$$

$$\mathbf{u}^{p+1} - \mathbf{u}^{p} = \frac{1}{\beta}\Big(\frac{\Delta t}{m}\nabla\cdot\boldsymbol{\sigma}^{p+1} + \frac{\Delta t}{m}\mathbf{R}^{p+1/2} \qquad\qquad\qquad \Big)$$

# New EVP equations

New momentum equations

$$\boldsymbol{\sigma}^{p+1} - \boldsymbol{\sigma}^{p} = \frac{1}{\alpha}\Big(\boldsymbol{\sigma}(\mathbf{u}^{p}) - \boldsymbol{\sigma}^{p}\Big),$$

$$\mathbf{u}^{p+1} - \mathbf{u}^{p} = \frac{1}{\beta}\Big(\frac{\Delta t}{m}\nabla\cdot\boldsymbol{\sigma}^{p+1} + \frac{\Delta t}{m}\mathbf{R}^{p+1/2} \boxed{+ \mathbf{u}_n - \mathbf{u}^{p}}\Big)$$

with $$\alpha\beta \gg \gamma = \frac{P}{2\Delta}\frac{(c\pi)^2}{A}\frac{\Delta t}{m}$$

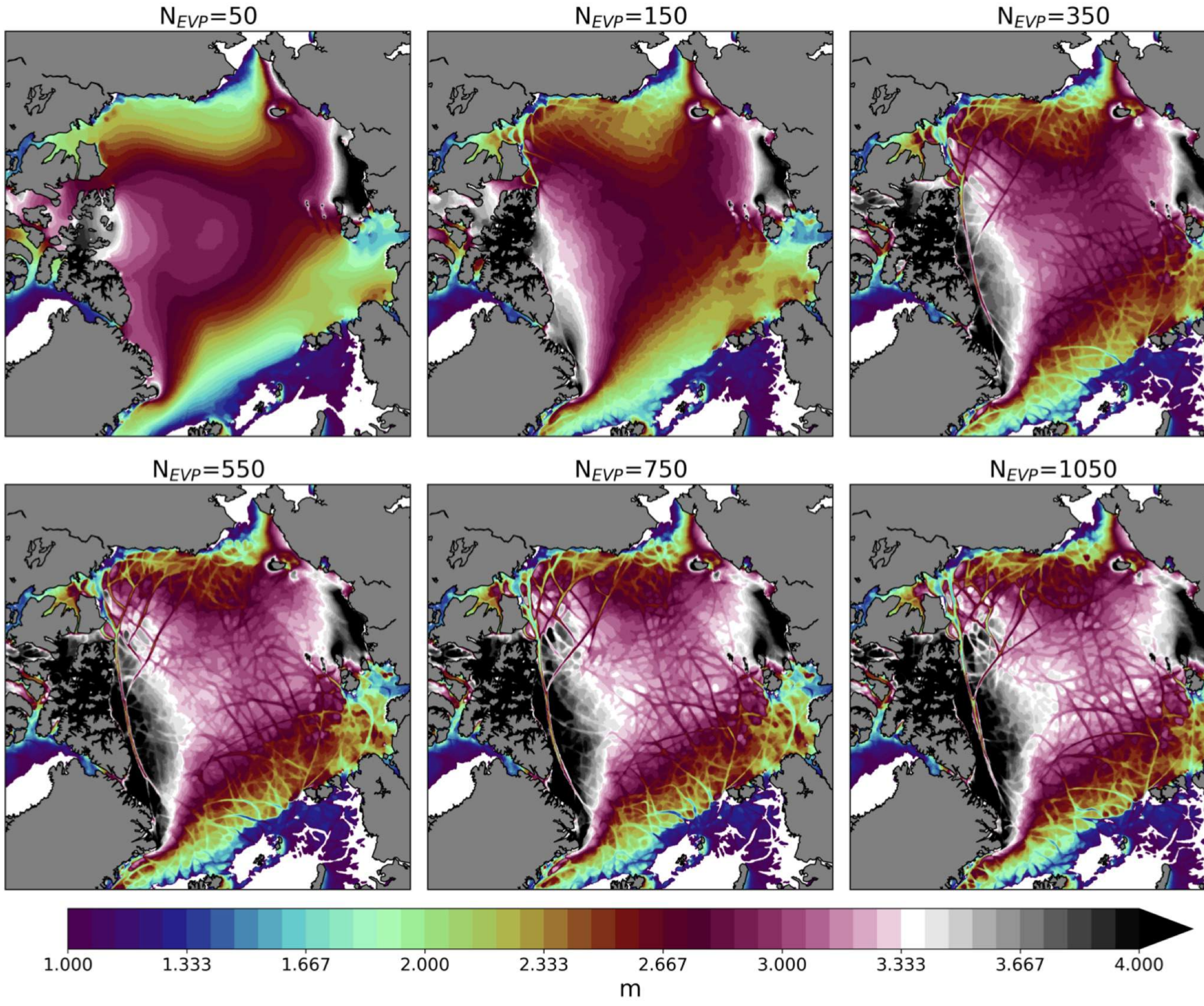
from stability analysis (Kimmritz et al, 2015, 2016).

modified EVP: $\alpha, \beta$ = constant, order(300)

adaptive EVP: $\alpha = \beta = (4\gamma)^{1/2}$

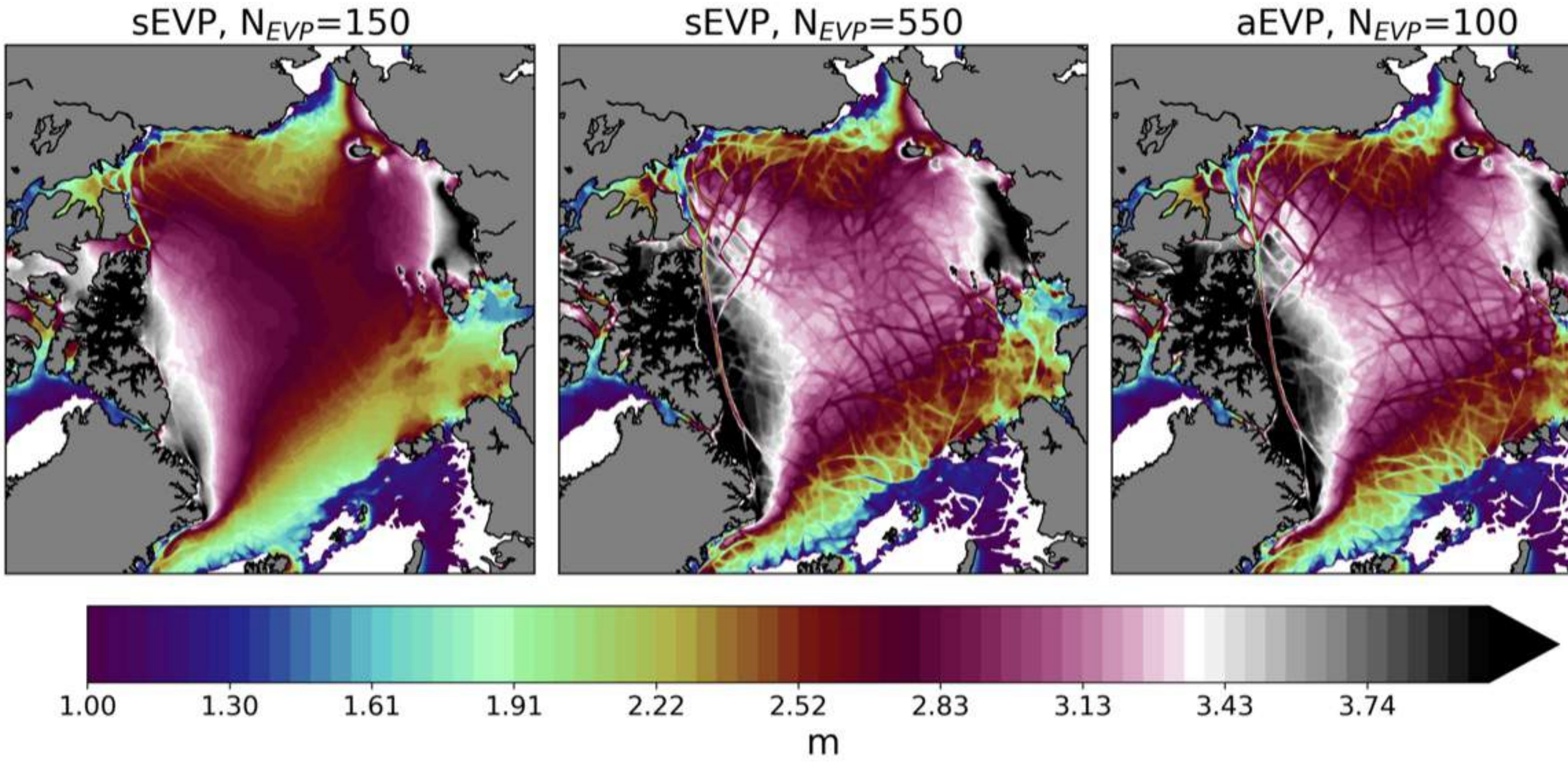
# high resolution simulations

# EVP “convergence” (in FESOM)

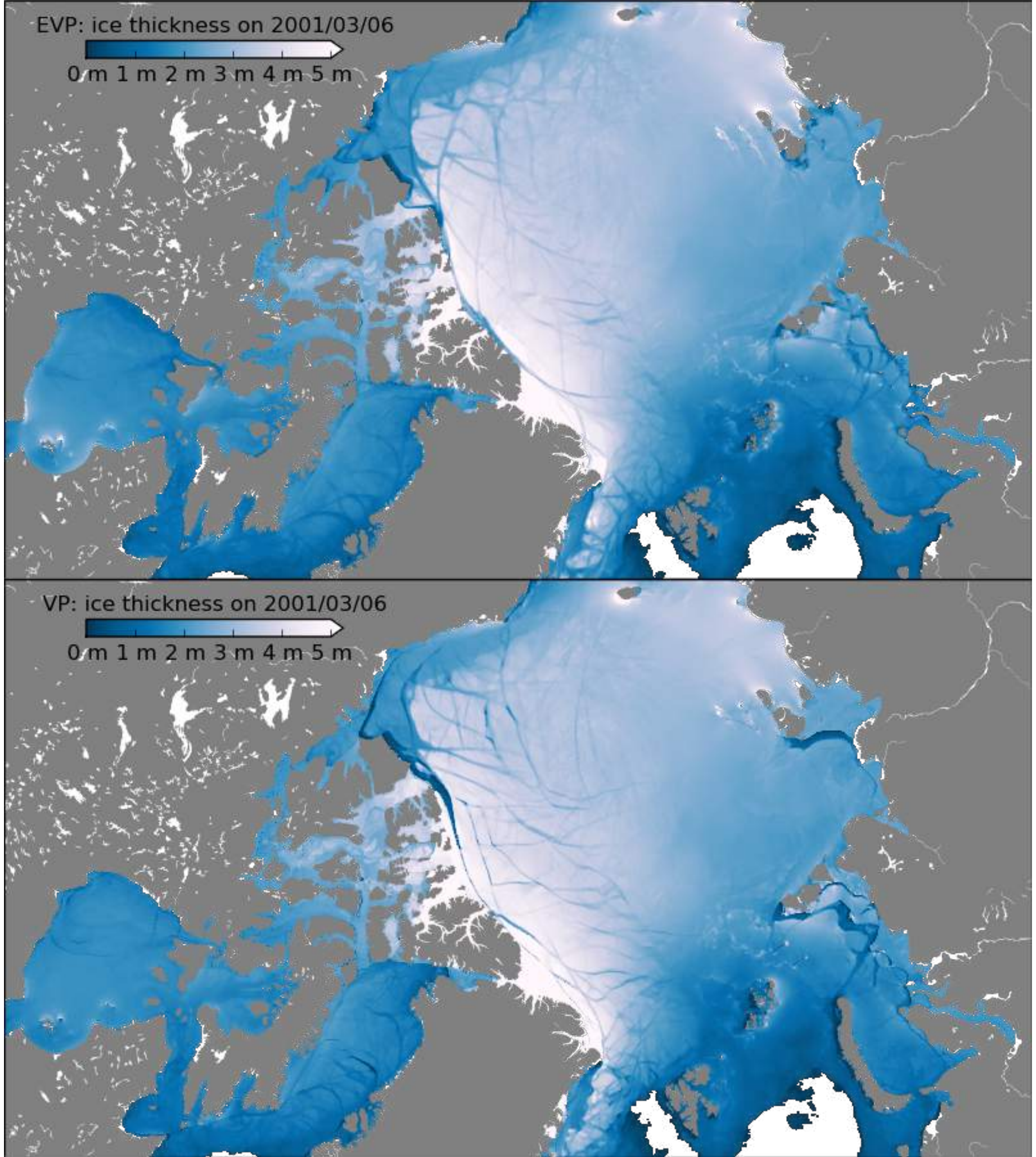



Koldunov et al. (2019), JAMES
~ 4 km

# EVP “convergence” (in FESOM)




Koldunov et al. (2019), JAMES, grid resolution ~ 4 km



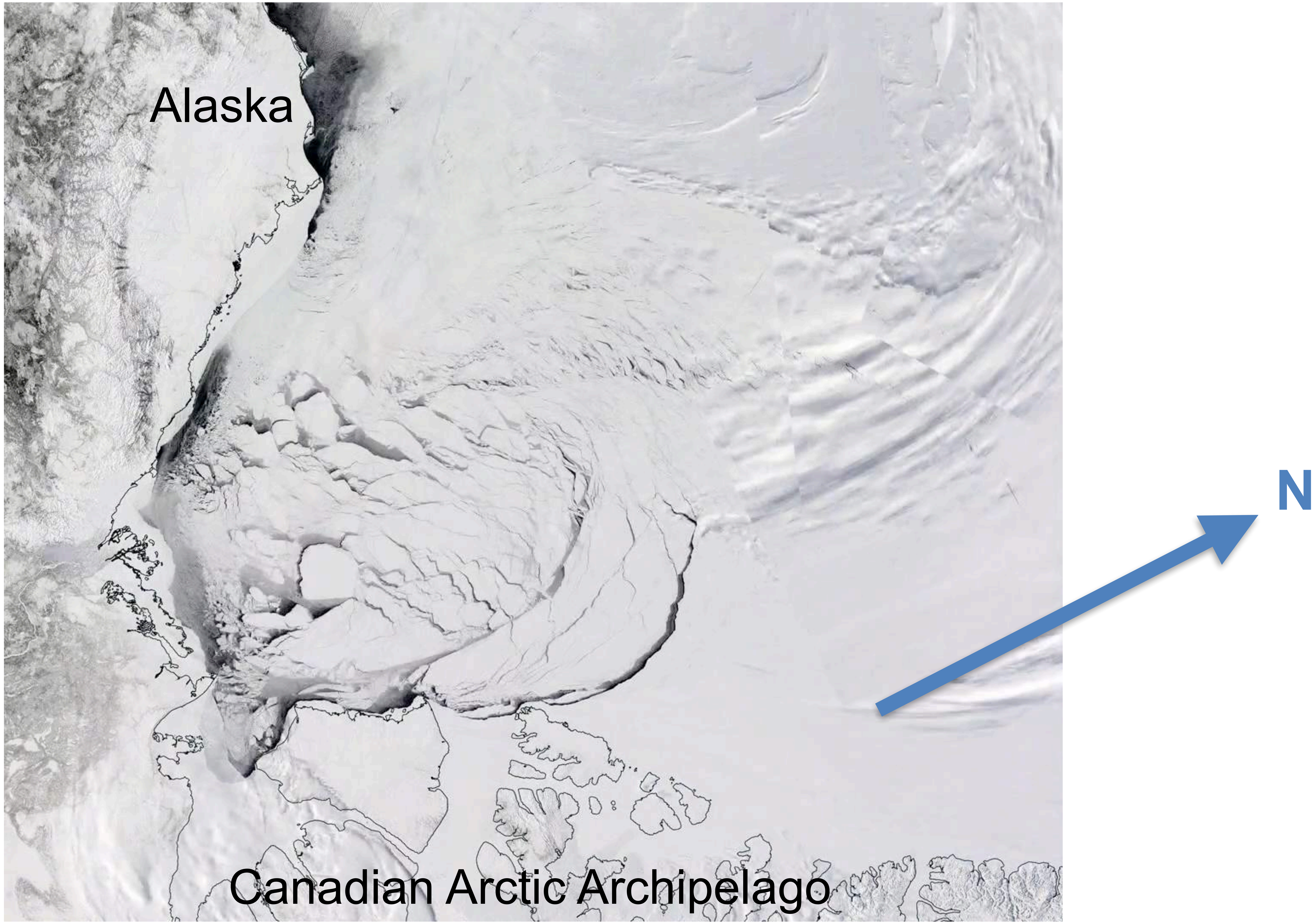

Convergence to VP solution:

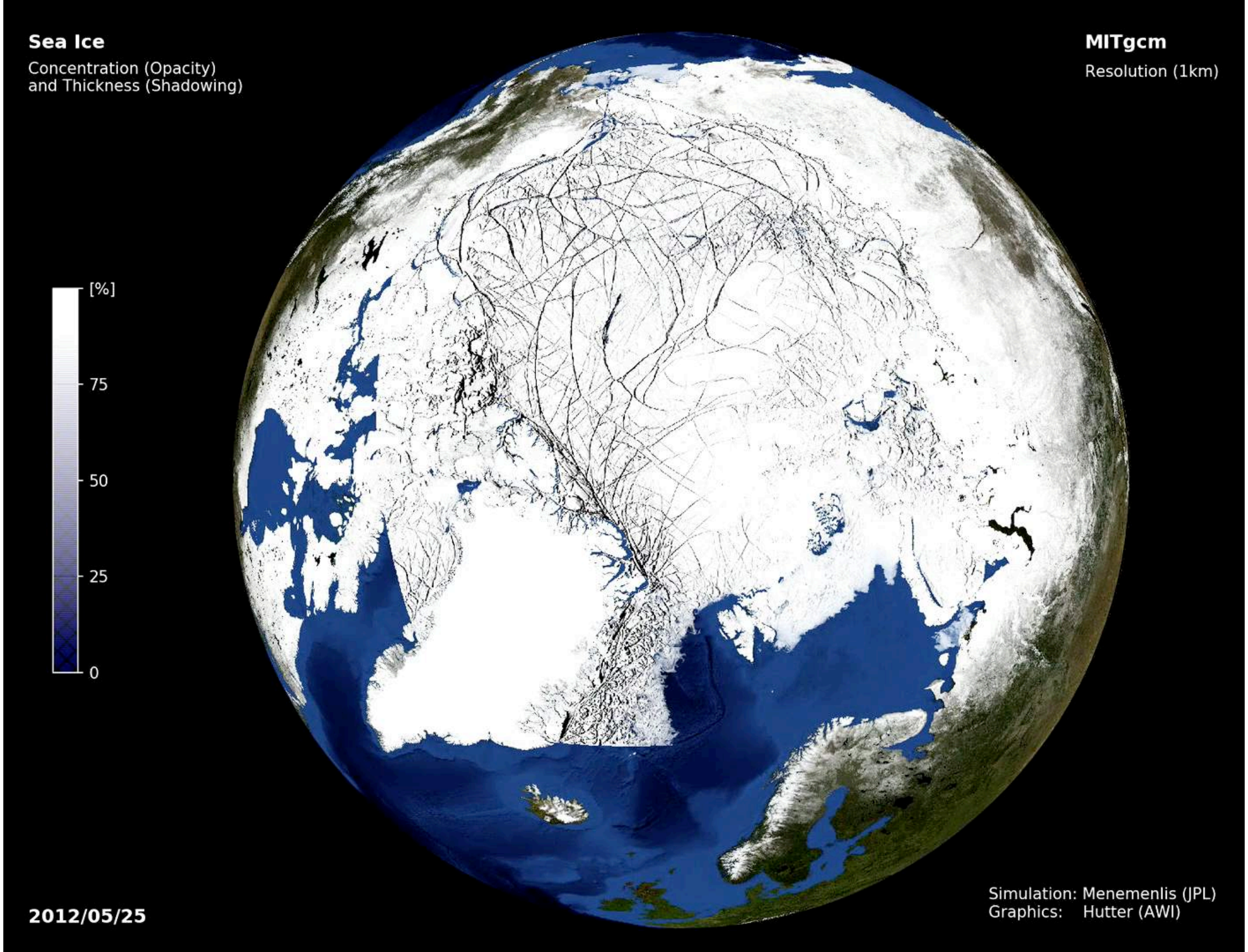
ice thickness (m) at 4.5 km grid spacing

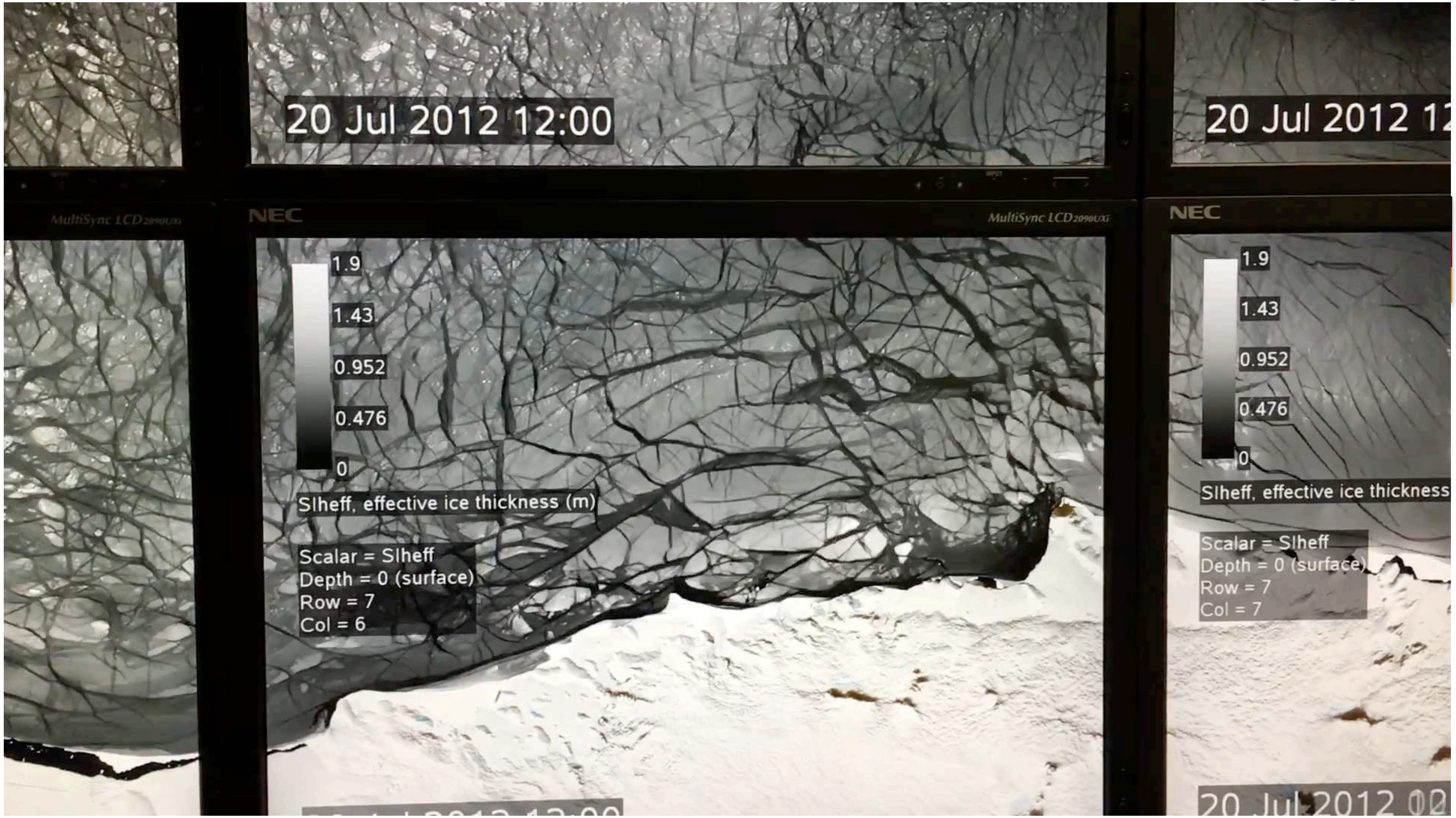
$$\alpha\beta \gg \gamma = \zeta \frac{(c\pi)^2}{A_c} \frac{\Delta t}{m}$$

stability parameter depends on grid spacing and local ice viscosity

# sea ice dynamics from optical satellite images



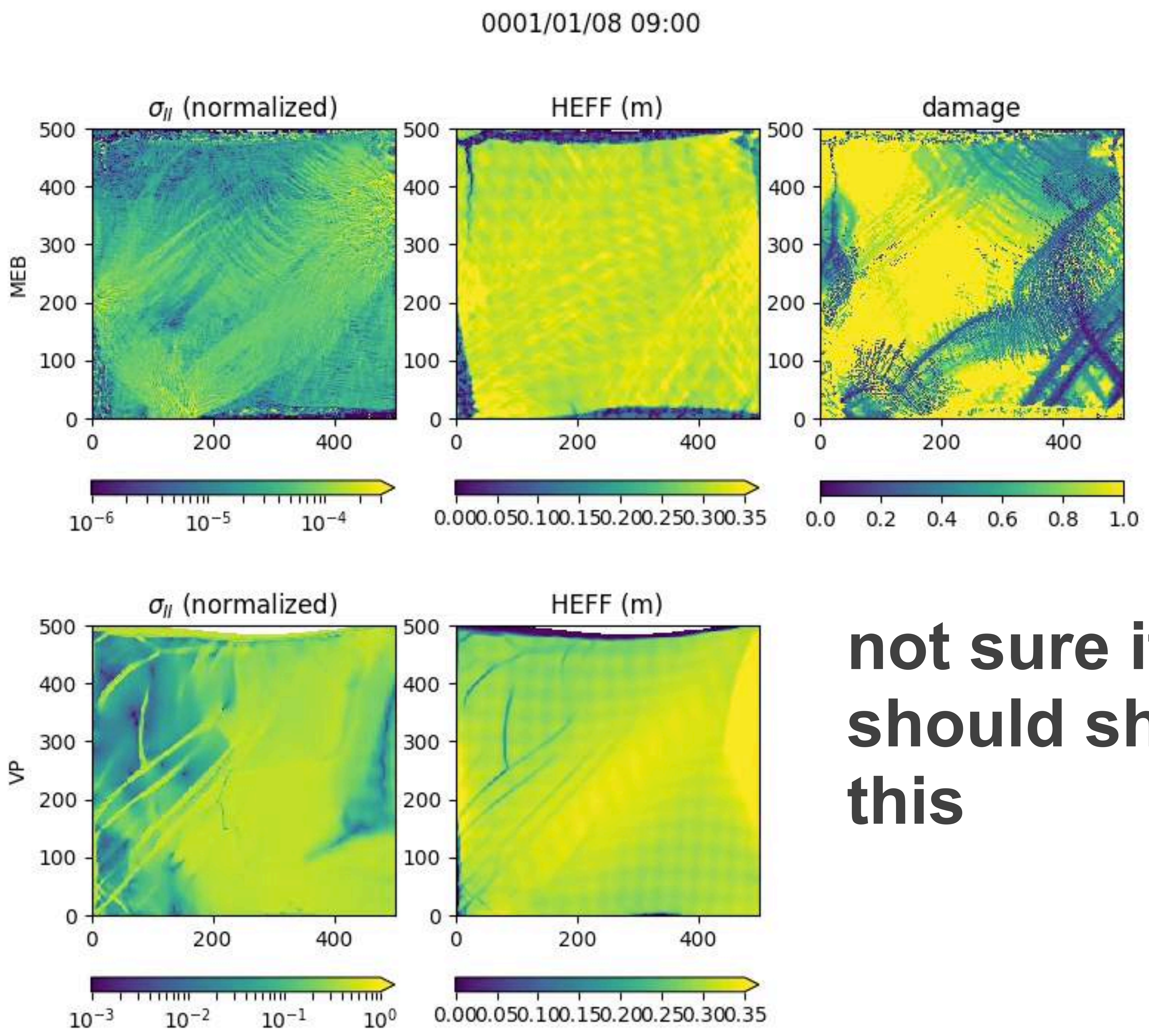

Sea Ice
Concentration (Opacity)
and Thickness (Shadowing)
MITgcm
Resolution (1km)
[%]
75
50
25
0
2012/05/25
Simulation: Menemenlis (JPL)
Graphics: Hutter (AWI)




Menemenlis, pers. comm

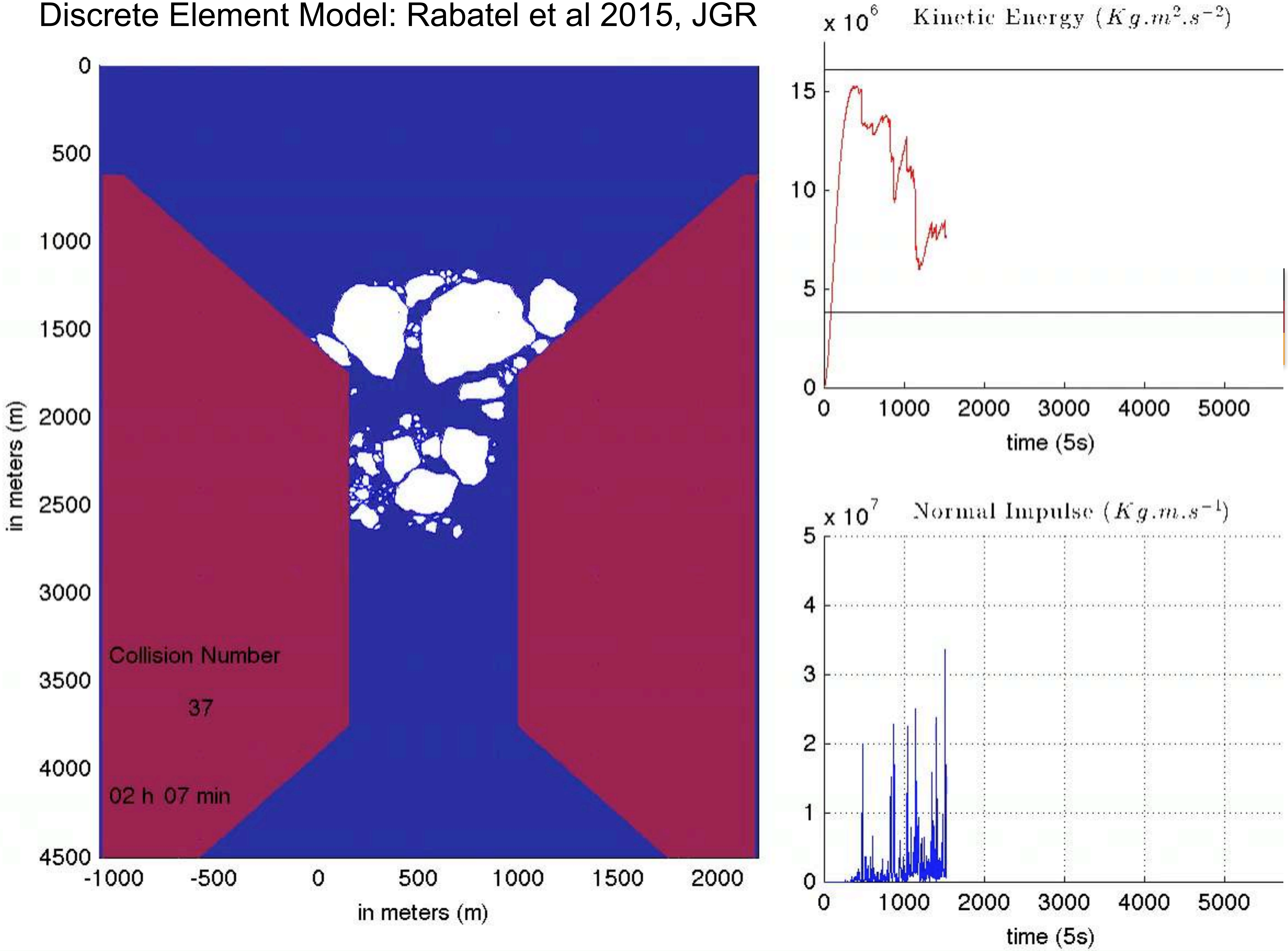
# Maxwell Elasto-Brittle rheology

- VP has been criticized for low intermittency and heterogeneity (Girard et al 2009, but Hutter et al 2018/2019 show opposite)
- Dansereau et al (2016): $$\frac{1}{E}\frac{\partial\sigma}{\partial t}+\frac{1}{\lambda}\sigma = K:\dot{\epsilon}$$
  - new model variable: damage (actually integrity of sea ice), affects ice strength
  - Mohr-Coulomb law for failure (increases damage) $\left(\tau = \mu\sigma + c\right)$
  - VP-viscosities are re-interpreted as constant coefficients; leads to a linear problem
- unclear: optimal solution strategy, computational cost (probably very high), stability, coupling to thermodynamics




**not sure if I should show this**

Discrete Element Model: Rabatel et al 2015, JGR



# Biogeochemistry in sea ice models

- ice BGC affects attenuation (uptake) of shortwave raditiation (albedo; modifies melt rate and availability to ocean), ocean BGC through “seeding” with biologically active material
- simple models for simple sea ice models, e.g. SIMBA (Castellani et al. 2017)
- multilayer models require more sophisticated sea ice models with positive definite vertical transport schemes, see documentation of “icepack” (column physics and biogeochemistry of CICE)
- numerous feedbacks need to be taken into account

# What's missing and where to go from here

- dynamics
  - explore continuity assumption at high resolution
  - new rheological approaches, anisotropy (EAP)
  - discrete element models for climate research?
  - surface and bottom stress (skin drag, form drag, ice roughness length, etc.)
  - porosity in ridges => ice strength parameterisations (Roberts et al. 2019)
- thermodynamics (most of this is in CICE/ICEPACK: https://github.com/CICE-Consortium)
  - multiple layers, vertical advection
  - melt pond parameterisations
  - snow parameterisations
  - biogeochemistry
- coupling to ocean and atmosphere
- …

# Literature about sea ice modelling


- Reviews:
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